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SIMPLIFIED EXPRESSIONS FOR THE GYROSYNCHROTRON RADIATION FROM MILDLY RELATIVISTIC, NONTHERMAL AND THERMAL ELECTRONS

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ABSTRACT

We present simplified expressions for the gyrosynchrotron radiation from mildly relativistic electrons, both for nonthermal (power-law) and for thermal (Maxwellian) distributions. We give expressions for the emissivity, absorptivity, effective temperature, frequency of peak brightness, and degree of circular polarization. The expressions are designed for the range from ~ 10 to ~ 100 of harmonic numbers, 20° to 80° of viewing angle θ, 3 to 7 electron energy spectral index (nonthermal electrons), and 10^8 to 10^9 K in temperature (thermal electrons). This range generally covers the cases of interest for the ~ 10 keV to ~ 1 MeV electrons that are probably important in solar and stellar flares and possibly in other astrophysical situations. The accuracy of our expressions, within the stated range of validity, is generally much better than a factor of 2, while the values of some of the quantities cover up to 15 orders of magnitude. The simplicity of the expressions should make them useful for semiquantitative investigations of models of astrophysical sources.

Subject headings: synchrotron radiation — Sun: radio radiation — Sun: flares — Stars: flares

I. INTRODUCTION

The purpose of this paper is to summarize theoretical results on the gyrosynchrotron radiation from mildly relativistic electrons, to compare the radiation from thermal and nonthermal distributions (Maxwellian and power-law), and to give simple approximations for the emission, absorption, and polarization which can be used in calculations of model radio sources. We are especially concerned with electrons of energies from ~ 10 keV to ~ 1 MeV, electrons which emit mainly between the 10th and 100th harmonic of the gyrofrequency and for which the simplifications of the nonrelativistic and ultra-relativistic limits are invalid. These electrons undoubtedly play a major role in producing the radio and hard X-ray emission of solar and stellar flares and are probably important in the vicinity of some shock waves within the corona and accretion flows onto dense objects such as neutron stars and white dwarfs.

In the case of solar flares, a controversy exists as to whether the bulk of the energy in the hard, impulsive phase of flares resides in thermal or nonthermal electrons (e.g., Ramaty *et al.* 1980). A possible way to resolve the controversy is to explore various models of the flaring source and to calculate the X-rays and microwaves emitted, first, if the electrons are thermal, and second, if nonthermal. Comparison of predictions of the two kinds of models with observations could then discriminate between the two hypotheses. But in order to calculate the radio emission from the model source, it is desirable to have relatively simple expressions for the gyrosynchrotron emission.

The general expressions for the emission and absorption coefficients of gyrosynchrotron emission are quite cumbersome (e.g., Ginzburg and Syrovatskii 1965; Melrose 1980, p. 98 ff.). For nonthermal, mildly relativistic electrons, numerical calculations have been applied (Ramaty 1969; Takakura and Scalise 1970). For thermal electrons, certain simplifications are possible (Trubnikov 1958), but even these simplified expressions are too cumbersome for some purposes. Therefore, we derive, in § III, approximate, power-law expressions for the gyrosynchrotron emissivity, absorptivity, and polarization for nonthermal electrons. In § IV, we give expressions for thermal electrons: slightly modified versions of formulae due to Petrosian (1981) and Dulk, Melrose, and White (1979) for the absorption coefficient, and a new expression for the polarization.

Overall, our simple expressions give surprisingly accurate results, especially for nonthermal electrons. Over the range of harmonic numbers, $\nu/\nu_B \approx 10$ to ≈ 100, electron power-law index $\delta \approx 2$ to ≈ 7, and viewing angle $\theta \approx 20°$ to 80°, the expressions are mostly accurate to much better than a factor of 2, even though the range of values covers some 15 decades. For thermal electrons, the same range of ν/ν_B and θ, but temperature $T < 10^8$ to $> 10^9$ K, the expressions are also mostly accurate to better than a factor of 2, and over about 20 decades.

In § II we set out the relations from radiative transfer in the form that is most useful for the interpretation of the radio emission, i.e., where measurements are made of brightness temperature, flux density, and spectrum of emission. In § III we give the approximate expressions for nonthermal (power-law) electrons and analyze their accuracy, and in § IV we give additional expressions for thermal (Maxwellian) electrons and analyze their accuracy.

II. RADIATIVE TRANSFER AND BRIGHTNESS TEMPERATURE

The equation of radiative transfer is usually written in terms of the specific radiation intensity I_ν and the source function $J_\nu = \eta_\nu/\kappa_\nu$ (see, for example, Chandrasekhar 1960), where η_ν and κ_ν represent the coefficients of emission and absorption (in units of ergs $\mathrm{cm^{-3}\ s^{-1}\ Hz^{-1}\ sr^{-1}}$ and $\mathrm{cm^{-1}}$ respectively). It is sometimes useful to employ a change of variables such that I_ν and J_ν are replaced by the brightness temperature, T_b, and the effective temperature of the radiating electrons, $T_{\rm eff}$, respectively. At radio wavelengths where the Rayleigh-Jeans law provides a good approximation to the Planck function, we can write:

$$I_\nu = kT_b \nu^2/c^2 \ , \tag{1}$$

$$J_\nu = kT_{\rm eff} \nu^2/c^2 \ . \tag{2}$$

In the case of a Maxwellian distribution of electrons, $T_{\rm eff}$ equals T irrespective of the emitting mechanism, frequency, and polarization mode; however, for a nonthermal distribution, $T_{\rm eff}$ in general is a function of both frequency and mode (e.g., Wild, Smerd, and Weiss 1963). Note that the usual factor of 2 is missing from equations (1) and (2) because we define I_ν and J_ν for each of the two orthogonal polarizations separately, and have:

$$I_\nu^{\rm tot} = I_\nu^{p_1} + I_\nu^{p_2} \ . \tag{3}$$

Equations (1) and (2) allow us to write the radiative transfer equation in the form

$$\frac{dT_b}{d\tau_\nu} = -T_b + T_{\rm eff} \ , \tag{4}$$

or, as illustrated in Figure 1,

$$T_b = \int_0^{\tau_\nu} dt_\nu\, T_{\rm eff}\, e^{-t_\nu} + T_{bo}\, e^{-\tau_\nu} \ . \tag{5}$$

These equations are valid only for media in which the density, n, is so low that the index of refraction is nearly unity. This situation, which applies in a wide range of astrophysical phenomena, is the only one for which the gyrosynchrotron formulae can be greatly simplified. In other situations, not considered here, medium suppression, i.e., the Razin-Tsytovich effect, must also be taken into account; specifically, suppression is important when $\nu < 20n/B$ (see, e.g., Ginzburg and Syrovatskii 1965).

In the special case of an isolated source with constant $T_{\rm eff}$, equation (5) reduces to:

$$T_b = T_{\rm eff}(1 - e^{-\tau_\nu}) \ . \tag{6}$$

$$\begin{aligned} T_b &= T_{\rm eff} && (\text{if } \tau_\nu \gg 1) \ , \\ &= T_{\rm eff}\,\tau_\nu = (c^2/k\nu^2)\eta_\nu L && (\text{if } \tau_\nu \ll 1) \ , \end{aligned} \tag{7}$$

where L is the dimension of the source along the line of sight. We note that the maximum value attainable by T_b is $T_{\rm eff}$.

In the following sections we give graphs and expressions for the emission and absorption coefficients of gyrosynchrotron radiation. The expressions are given as functions of the "harmonic number" ν/ν_B, where $\nu_B = 2.8 \times 10^6\ B$ (Hz) is the gyrofrequency and B is the magnetic field strength (G). To make these results general, we note that

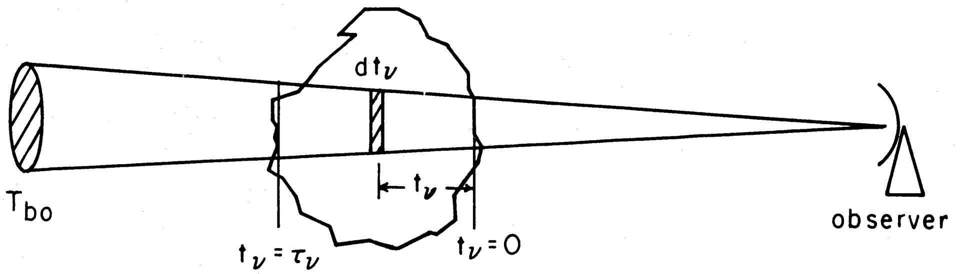

FIG. 1.—Geometry for eq. (5)

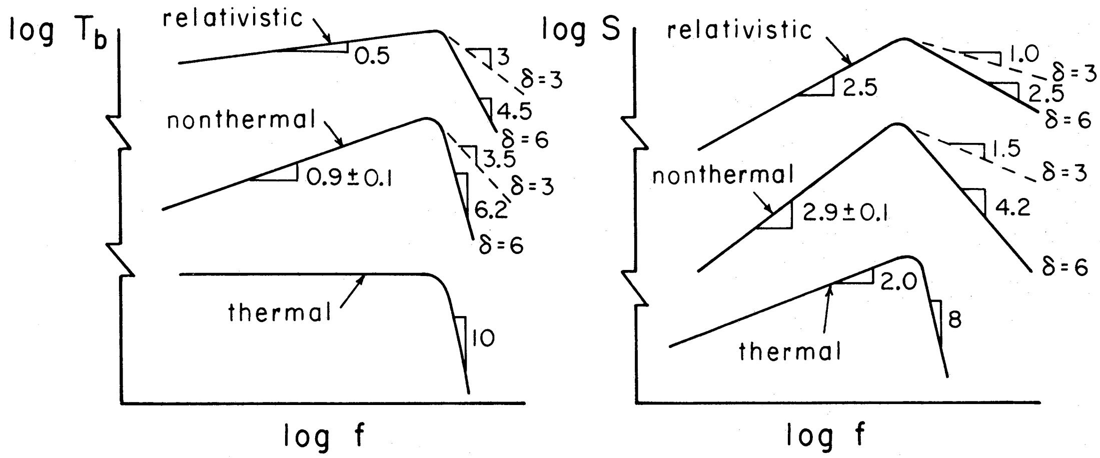

FIG. 2.—Shapes of brightness and flux spectra for thermal and nonthermal electron distributions. The shapes of the curves for relativistic power-law distributions were taken from Ginzburg and Syrovatskii (1965).

$\eta_\nu \propto NB$ and $\kappa_\nu \propto N/B$, where N is the total number of electrons per cm^3 above some energy E_o (in the case of a Maxwellian distribution, $E_o = 0$ and $N = n$, the number density). Thus, it is convenient to work with the quantities η_ν/NB and $\kappa_\nu B/N$. In terms of these quantities, T_{eff} becomes

$$T_{\text{eff}} = \frac{c^2}{k\nu^2}\frac{\eta_\nu}{\kappa_\nu} = \frac{c^2}{k}\frac{(\eta_\nu/BN)}{(\kappa_\nu B/N)}\frac{B^2}{\nu^2} = 8.33 \times 10^{23}\frac{(\eta_\nu/BN)}{(\kappa_\nu B/N)}\left(\frac{\nu}{\nu_B}\right)^{-2} . \tag{8}$$

The flux density S (for one polarization) of a radio source is related to the brightness temperature by the relation

$$S = \frac{k\nu^2}{c^2}\int d\Omega T_b , \tag{9}$$

where $d\Omega$ is a differential solid angle, and the integral is over the projected area of the source.

The polarization of the radiation is, in general, described by radiative transfer equations involving polarization tensors (see, e.g., Melrose 1980, p. 196); however, here we will discuss only the simple case of the circular polarization of radiation from a homogeneous plasma in which the characteristic modes (the o-mode and x-mode) are circular. Specifically, we exclude the case of propagation nearly perpendicular to the magnetic field, i.e., the case when $|\pi/2 - \theta| < \nu_B/2\nu$. Under these conditions the degree of circular polarization r_c is:

$$r_c = (T_{b,x} - T_{b,o})/(T_{b,x} + T_{b,o}) . \tag{10a}$$

From our numerical calculations we find that, if $\tau \gg 1$ for both modes, then the polarization is in the sense of the o-mode, $|r_c| \leq 0.2$ for all $\theta \gtrsim 20°$ and $|r_c| \leq 0.08$ for $\theta \geq 60°$ ($r_c = 0$ for thermal electrons); these small degrees of polarization are not easily measured or interpreted, and we will not consider this case further. More important is the case when $\tau \ll 1$ for both modes; our results below apply to this case. Here we can replace equation (10a) by

$$r_c = (\eta_{\nu,x} - \eta_{\nu,o})/(\eta_{\nu,x} + \eta_{\nu,o}) , \qquad (\tau_\nu \ll 1) . \tag{10b}$$

Two cautionary remarks are necessary. First, for equation (10b) to be valid, there can be no significant variations in plasma properties along the line of sight within the emission region, i.e., no significant changes in B, θ, or T_{eff}. Second, as defined, the polarization applies to brightness temperature, not to flux density. The latter involves an integration not only along the line of sight but over all parts of the source; therefore, if B, θ, or T_{eff} vary with position in the source, as frequently is the case, then the polarization of flux density is the average (weighted by T_b) of the polarization of brightness temperature.

Another useful quantity for diagnostics is ν_{peak}, the frequency of peak microwave flux density as depicted in Figure 2. The peak in the spectrum occurs at that frequency where $\tau_\nu = \kappa_\nu L \approx 1$. It turns out that ν_{peak} depends most sensitively on the magnetic field strength so that, if estimates of electron energies and numbers are available from other sources, such as hard X-rays, then the field strength can be found.

III. GYROSYNCHROTRON EMISSION FROM NONTHERMAL ELECTRONS

We now consider a plasma in which the emitting electrons have an isotropic pitch angle distribution and a power-law distribution in energy:

$$n(E) = KE^{-\delta} , \tag{11}$$

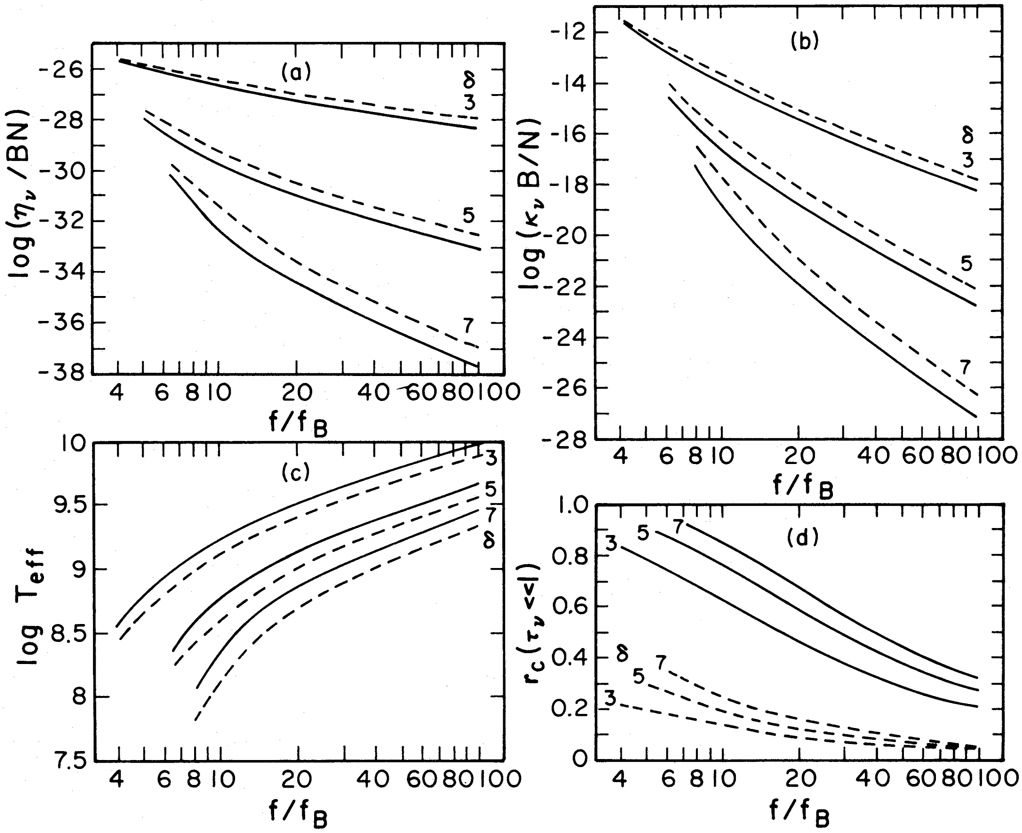

FIG. 3.—Characteristics of gyromagnetic emission for the x-mode calculated numerically from the formulae of Takakura and Scalise (1970). Curves are given for three values of electron power-law index δ and viewing angles 40° (*solid lines*) and 80° (*dashed lines*). The low-energy cutoff in the electron distribution is 10 keV. Shown are: (*a*) emission coefficient, (*b*) absorption coefficient, (*c*) effective temperature, and (*d*) degree of circular polarization.

where K is related to N, the number of electrons per cm^3 with $E > E_o$, by the relation

$$K = (\delta - 1)E_o^{\delta-1}N\ . \tag{12}$$

We henceforth assume that the low-energy cutoff is $E_o = 10$ keV $= 1.6 \times 10^{-8}$ ergs. (We have also made calculations assuming $E_o = 30$ keV and, except for the normalization in eq. [12], the results are identical to those given below.)

Our numerical calculations were made using the formulae for gyrosynchrotron emission and absorption given by Takakura and Scalise (1970) and Takakura (1972). These formulas are valid for media where deviations of the index of refraction from unity can be ignored, in particular, when Razin suppression is unimportant.

Figures 3*a*, 3*b*, 3*c*, and 3*d*, respectively, show η_ν/BN, $\kappa_\nu B/N$, $T_{\rm eff}$, and r_c as a function of ν/ν_B. On each figure, curves are given for three values of electron energy spectral index δ and two values of viewing angle θ. We see that the slopes of the curves are approximately constant for $\nu/\nu_B \geq 10$. We are thus able to derive power-law expressions which, over the frequency range of interest, are good approximations for each of these quantities and for $\nu_{\rm peak}$. We note that only the first two of these quantities are independent, for the others are derivable from equations (2) and (10). However, all are useful, for η_ν/BN alone is needed if $\tau_\nu \ll 1$, $T_{\rm eff}$ alone if $\tau_\nu \gg 1$, and $\kappa_\nu B/N$ or $\nu_{\rm peak}$ alone if $\tau_\nu \approx 1$. After some experimentation, we find the following expressions are reasonable approximations for the x-mode and for $2 \leq \delta \leq 7$, $\theta \geq 20°$, and $\nu/\nu_B \geq 10$:

$$\frac{\eta_\nu}{BN} \approx 3.3 \times 10^{-24}10^{-0.52\delta}(\sin\theta)^{-0.43+0.65\delta}\left(\frac{\nu}{\nu_B}\right)^{1.22-0.90\delta}, \tag{13}$$

$$\frac{\kappa_\nu B}{N} \approx 1.4 \times 10^{-9}10^{-0.22\delta}(\sin\theta)^{-0.09+0.72\delta}\left(\frac{\nu}{\nu_B}\right)^{-1.30-0.98\delta}, \tag{14}$$

$$T_{\rm eff} \approx 2.2 \times 10^9 10^{-0.31\delta}(\sin\theta)^{-0.36-0.06\delta}\left(\frac{\nu}{\nu_B}\right)^{0.50+0.085\delta}, \tag{15}$$

$$r_c \approx 0.20 \times 10^{0.05\delta} 10^{1.93 \cos\theta - 1.16 \cos^2\theta} \left(\frac{\nu}{\nu_B}\right)^{-0.21 - 0.37 \sin\theta} \quad (\text{if } \tau_\nu \ll 1)\,, \tag{16}$$

$$\nu_{\text{peak}} \approx 2.72 \times 10^3 10^{0.27\delta} (\sin\theta)^{0.41 + 0.03\delta} (NL)^{0.32 - 0.03\delta} B^{0.68 + 0.03\delta}\,. \tag{17}$$

The accuracy of our approximation for the absorption coefficient can be judged from Figure 4, which, for $\theta = 20°$, 45°, and 80°, shows contours of the error:

$$\epsilon = \log\left[\frac{\text{approximate value}}{\text{true value}}\right], \tag{18}$$

where the "true" values were calculated numerically from the full expressions. Within the range of δ and ν/ν_B of greatest interest, our approximation is generally better than $|\epsilon| = 0.1$, i.e., an error of 26%, but outside this range, the error increases rapidly. Also, the errors shown in Figure 4 are representative of those in our approximations for η_ν/BN, ν_{peak}, and T_{eff}. For r_c the percentage errors are even smaller, as seen in Figure 5.

IV. GYROSYNCHROTRON EMISSION FROM THERMAL ELECTRONS

We turn now to a plasma in which the emitting electrons have an isotropic pitch angle distribution and a Maxwellian distribution in energy:

$$u(\gamma)d\gamma = \frac{\pi}{2}\left(\frac{2mc^2}{\pi kT}\right)^{3/2}\left(1 + \frac{15kT}{8mc^2}\right)^{-1} \gamma(\gamma^2 - 1)^{1/2} \exp\left[-\frac{(\gamma - 1)mc^2}{kT}\right] d\gamma\,, \tag{19}$$

where γ is the Lorentz factor. Except for the normalization factor, this equation is valid for all T; the form of the

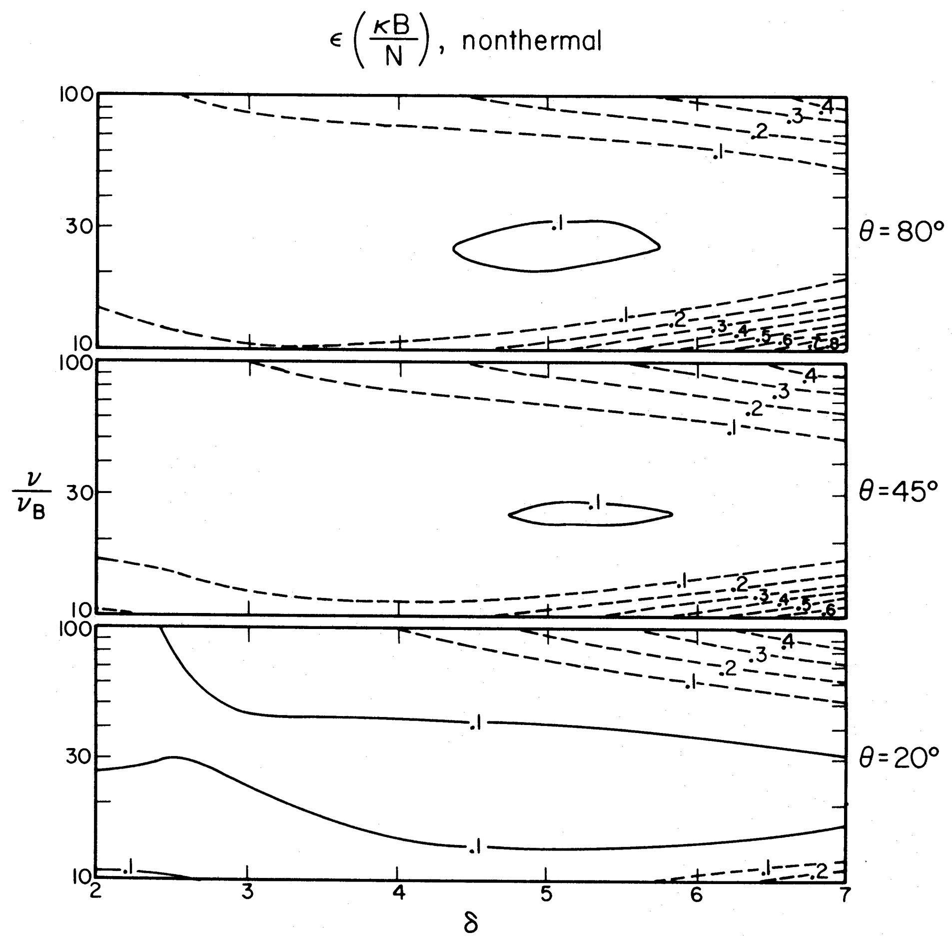

FIG. 4.—Errors involved in our approximate expression (eq. [14]) for $\kappa_\nu B/N$ of nonthermal electrons of energy index δ. The contours represent the error as defined by eq. (18). Full lines indicate where the approximation gives values which are too large, dashed lines where too small. The contours at the top are for $\theta = 80°$, in the middle for $\theta = 45°$, and at the bottom for $\theta = 20°$.

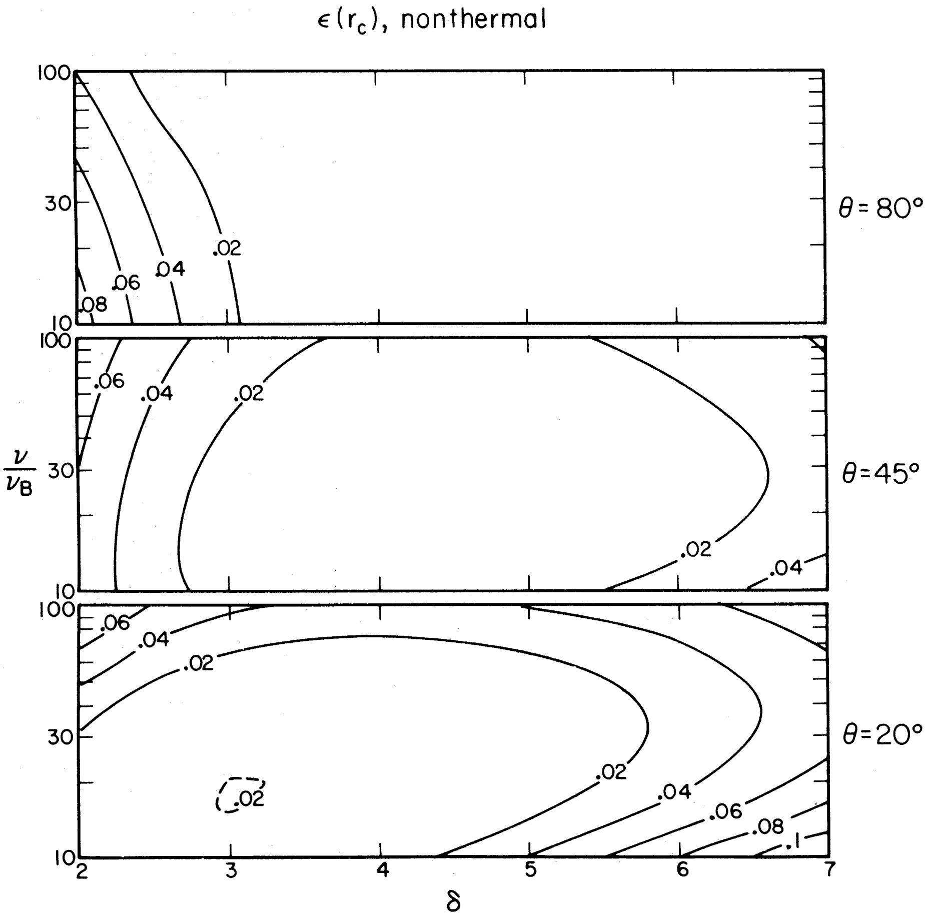

FIG. 5.—Same as Fig. 4 but for the degree of circular polarization of radiation from nonthermal electrons

normalization factor used here includes the leading terms of the expression of the modified Bessel function K_2 and is accurate for $kT < mc^2$ or $T < 6 \times 10^9$ K.

With Maxwellian electrons there are fewer independent quantities than for nonthermal electrons, i.e., we have the relation $T_{\rm eff} = T$, or equivalently, the emissivity is related to the absorptivity by Kirchoff's law:

$$\eta_\nu = \kappa_\nu kT \frac{\nu^2}{c^2} . \tag{20}$$

Curves showing the absorption coefficient for thermal electrons in a form similar to Figure 4 were given by Dulk, Melrose, and White (1979, hereafter DMW), so we will not repeat them here. Those curves have a distinctly non-constant slope, thus an accurate analytical expression for κ_ν must be more complicated than the one derived above for nonthermal electrons. Petrosian (1981) has derived an expression for the emissivity of thermal electrons (also for nonthermal electrons, but we do not utilize it here). We find that his expression for thermal electrons is quite accurate and has quite a wide range of validity—wider, for example, than the expressions of Trubnikov (1958). We give a slightly modified version as equation (21a), together with the considerably simpler but less accurate expression (eq. [21b]) from DMW. Each expression is likely to have its uses; for example, equation (21b) is useful at times when semiquantitative analytical modeling is called for, and equation (21a) when more accurate numerical modeling is warranted. For the degree of circular polarization (not obtainable from Petrosian's work), we have derived the empirical expression given in equation (23). For $\nu_{\rm peak}$, we have not been able to improve significantly on the expression (eq. [24a] below) from DMW, an expression which is substantially the same as one (eq. [24b] but where we have added the term in sin θ) derived by Wada *et al.* (1980) based on numerical computations by Chanmugam and Wagner (1979); equation (24a) is most accurate for $10^8 < T < 10^9$ K, while equation (24b) is most accurate for $10^7 \lesssim T \lesssim 10^8$ K (Wada *et al.* claim errors less than 20% when $\theta \approx 90°$).

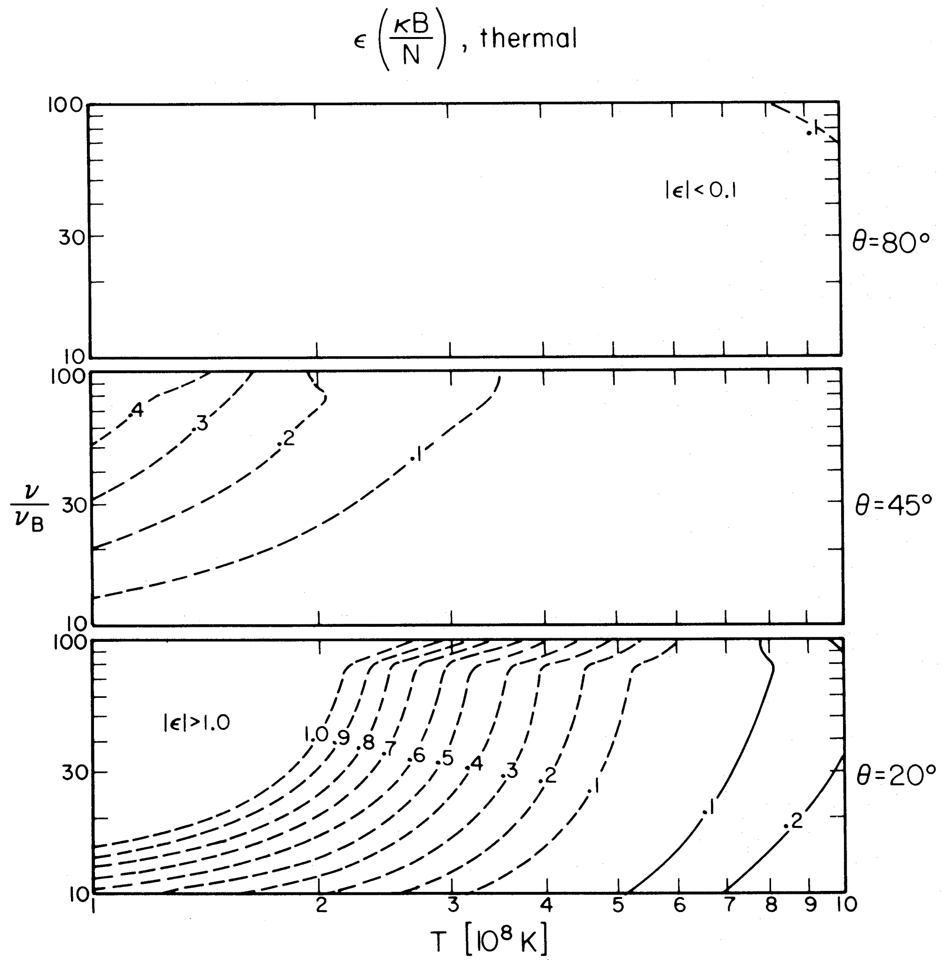

FIG. 6.—Same as Fig. 4 but for the absorption coefficient of radiation from *thermal* electrons. Our modified version, eq. (21a), of Petrosian's (1981) expression was used. Our version leaves out Petrosian's term $\gamma_o^{1/2}(1 - \beta_o^2 \cos^2 \theta)^{1/4}$ and is larger by a factor of $4/\sin\theta$.

The approximate expressions are, for thermal electrons:

$$\frac{\kappa_\nu B}{N} \approx 4.2 \times 10^{-9}(\sin\theta)^{-1}\left(1 + \frac{2\cot^2\theta}{\gamma_o^2}\right)\left(\frac{kT}{mc^2}\right)^{-2}\left(1 + \frac{15kT}{8mc^2}\right)^{-1}(\gamma_o^2 - 1)$$

$$\times\,(3\gamma_o^2 - 1)^{-1/2}\left(\frac{\nu}{\nu_B}\right)^{-3/2}\exp\left[-\frac{(\gamma_o - 1)mc^2}{kT}\right]Z_{\max}^{2\mu}(t_o)\,, \tag{21a}$$

where

$$\gamma_o^2 = 1 + 2\frac{kT}{mc^2}\frac{\nu}{\nu_B}\left(1 + 4.5\frac{kT}{mc^2}\frac{\nu}{\nu_B}\sin^2\theta\right)^{-1/3},$$

$$t_o^2 = (\gamma_o^2 - 1)\sin^2\theta\,, \qquad \mu = \frac{1 + t_o^2}{\gamma_o}\frac{\nu}{\nu_B}\,, \qquad Z_{\max}(t_o) = \frac{t_o \exp(1 + t_o^2)^{-1/2}}{1 + (1 + t_o^2)^{1/2}}\,,$$

$$\frac{\kappa_\nu B}{N} \approx 50T^7 \sin^6\theta B^{10}\nu^{-10}\,, \tag{21b}$$

$$\frac{\eta_\nu}{BN} \approx 1.2 \times 10^{-24}T\left(\frac{\nu}{\nu_B}\right)^2\frac{\kappa_\nu B}{N}\,, \tag{22}$$

$$r_c \approx 6.1T^{-0.18}10^{2.1\cos\theta - 1.3\cos^2\theta}\left(\frac{\nu}{\nu_B}\right)^{0.045 - 0.30\sin\theta} \qquad (\tau_\nu \ll 1)\,, \tag{23}$$

$$\nu_{\text{peak}} \approx \begin{cases} 1.4\left(\dfrac{NL}{B}\right)^{0.1} (\sin\theta)^{0.6} T^{0.7} B & (10^8 < T < 10^9 \text{ K}), \quad (24a) \\ 475\left(\dfrac{NL}{B}\right)^{0.05} (\sin\theta)^{0.6} T^{0.5} B & (10^7 < T < 10^8 \text{ K}). \quad (24b) \end{cases}$$

Figure 6 shows contours of the errors involved in the expression for $\kappa_\nu B/N$ in equation (21a). Again, they are generally better than $|\epsilon| \leq 0.1$, i.e., 26%. For r_c the percentage errors are shown in Figure 7; they are small, considering the relative simplicity of equation (23) compared with equation (21), but are not as small as for the nonthermal case.

V. DISCUSSION

Figure 2 shows schematic drawings of brightness temperature and flux spectra of a simple, isolated source. On the low-frequency (optically thick) side of the peak, the spectra are steeper for nonthermal than for thermal distributions. The fact that observed flux spectra of solar flares commonly have a slope of about +1, i.e., less steep than any theoretical spectrum, is therefore a direct indication that actual flare source sizes decrease with frequency. The rate of decrease with frequency would, for a given source model, have to be considerably larger for a nonthermal electron distribution than for a thermal one.

On the high-frequency (optically thin) side of the peak, the spectra are considerably less steep for nonthermal than for thermal distributions, reflecting the contributions of the highest energy electrons, which are more numerous in power-law than in Maxwellian distributions. Observed spectra in the earliest phases of impulsive flares often have a steep slope at high frequencies, possibly indicative of a thermal distribution, but soon thereafter the slope usually becomes less steep, probably indicating that a nonthermal tail exists, enhancing the numbers of electrons at high energies.

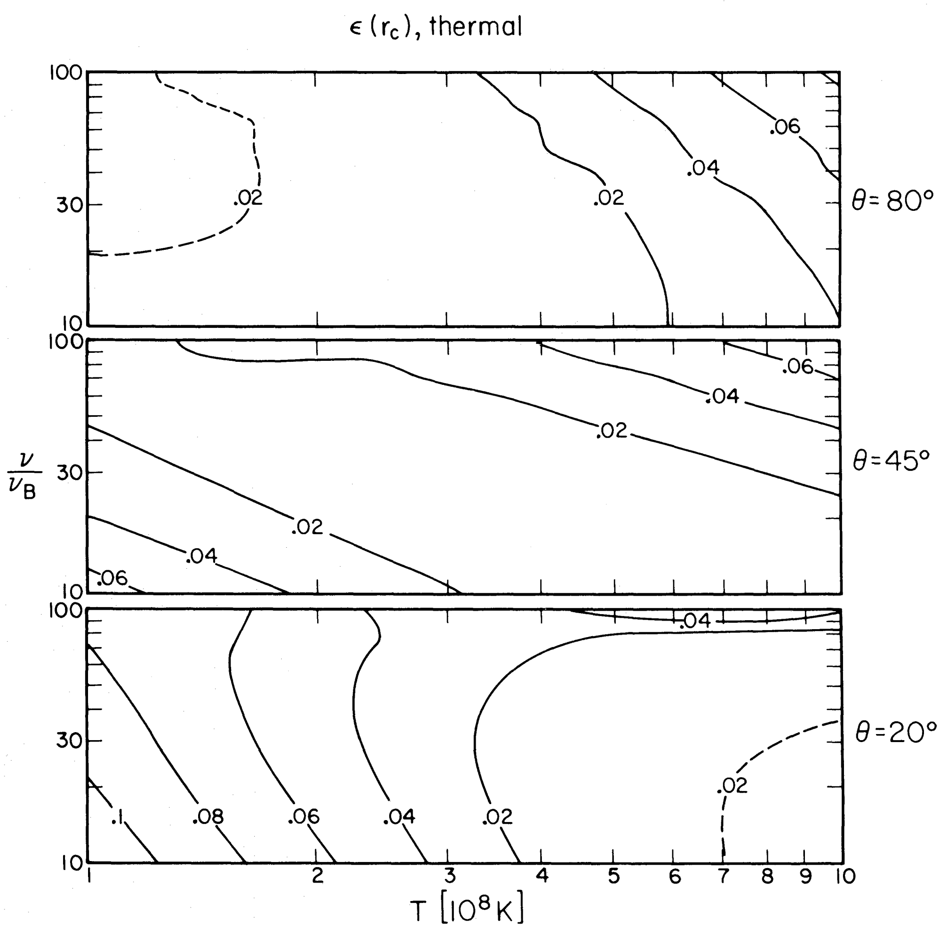

FIG. 7.—Same as Fig. 4 but for the degree of circular polarization of radiation from *thermal* electrons

It is interesting to note in Figure 3*c* that the values of $T_{\rm eff}$ for nonthermal electrons mostly lie between 10^8 K and 10^{10} K and that the largest values are attained for small δ, large ν/ν_B, and small θ. Because T_b reaches $T_{\rm eff}$ only when $\tau_\nu \gg 1$, this implies that T_b at a given frequency will be large only when (*a*) there are large numbers of nonthermal electrons along the line of sight (to make τ_ν large) and (*b*) when the magnetic field strength is low (requiring a high harmonic and hence high-energy electrons) or its direction is at a small angle to the line of sight (a circumstance when only the higher energy electrons emit significantly). Under solar conditions, field strengths are seldom very small and for frequencies of interest, ν/ν_B is generally ≈ 10 to ≈ 50. Thus, it is difficult or impossible to explain brightness temperatures larger than a few times 10^9 K in terms of gyrosynchrotron emission from a collection of individual, nonthermal electrons. When sources much brighter than 10^9 K are observed, a collective mechanism such as masering or plasma radiation is indicated.

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