(Solar) MHD and shock waves

University of Turku
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Doc. Rami Vainio
Department of Physics
University of Helsinki, Finland

rami.vainio@helsinki.fi

Outline

- Solar magnetohydrodynamics
  - MHD equations
  - MHD waves
- MHD shock waves
  - Basic concepts
  - MHD jump conditions
  - Classification of shocks
  - Coronal and interplanetary shocks
- Particle acceleration at shocks
  - Solar energetic particles
  - Acceleration mechanisms: drift, diffusive, surfing

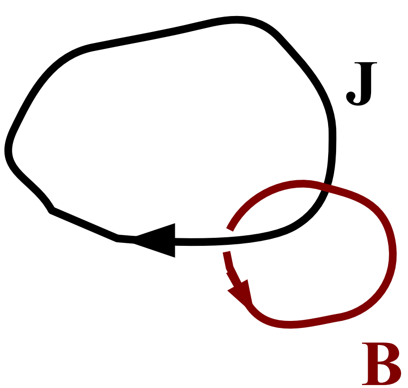
Magnetohydrodynamics (MHD)

- Basic building blocks:
  - Maxwell's eqs.; fluid equations; Ohm's law
- Maxwell:

$$
\begin{aligned}
\nabla \cdot \mathbf{D} &= \rho_q \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} .
\end{aligned}
\quad \longrightarrow \quad \frac{\partial \rho_q}{\partial t} + \nabla \cdot \mathbf{J} = 0
$$

conservation of charge

- In MHD, $\mathbf{B} = \mu_0 \mathbf{H}$
- In MHD, displacement current $\partial \mathbf{D} / \partial t$ neglected
  - $\nabla \cdot \mathbf{D} = \rho_q$ unnecessary
  - Ampère's law, $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, sufficient $\rightarrow \nabla \cdot \mathbf{J} = 0$



Ohm's law

- In a frame, co-moving with the plasma, **Ohm's law** reads

$$\mathbf{J} = \sigma \mathbf{E}'$$

- **Conductivity** σ of the plasma different in directions || and $\perp$ to the magnetic field $\rightarrow$ tensor quantity
- In MHD, σ is usually **approximated with a scalar**
- Lorentz transformation of the electric field:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v}\times\mathbf{B}; \qquad \mathbf{v} = \text{plasma flow velocity past the observer}$$

- Thus, Ohm's law in the frame of the observer is

$$\boxed{\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B})}$$

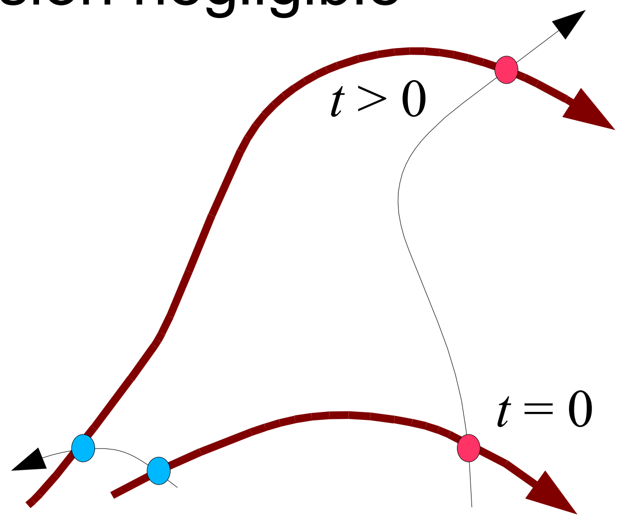
- In space plasmas, conductivity σ is often very large $\rightarrow$ **Ideal Ohm's law**

$$\boxed{\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0}$$

Ideal MHD

- v non-zero and large conductivity → diffusion negligible
- Expressed as $R_m >> 1$
- Thus, Faraday + Ideal Ohm's law →

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$



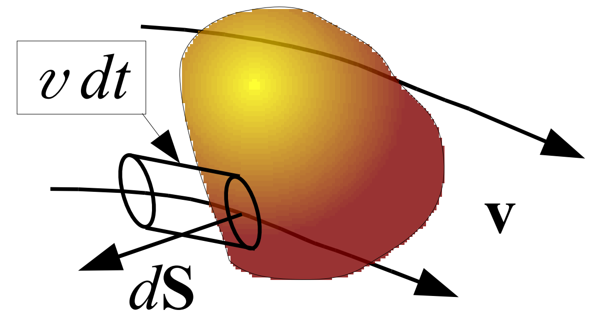
- Field lines become **frozen-in** into to the plasma flow:
  - two plasma parcels initially connected by a magnetic field line remain magnetically connected forever
  - strong constraint
  - dynamical phenomena often connected with non-ideal phenomena, i.e., breaking of frozen-in condition

Fluid equations

- Conservation of mass

$$\int_V \frac{\partial \rho}{\partial t} dV = -\oint_{\partial V} \rho \mathbf{v} \cdot d\mathbf{S} = -\int_V \nabla \cdot (\rho \mathbf{v})\, dV \;\Rightarrow$$

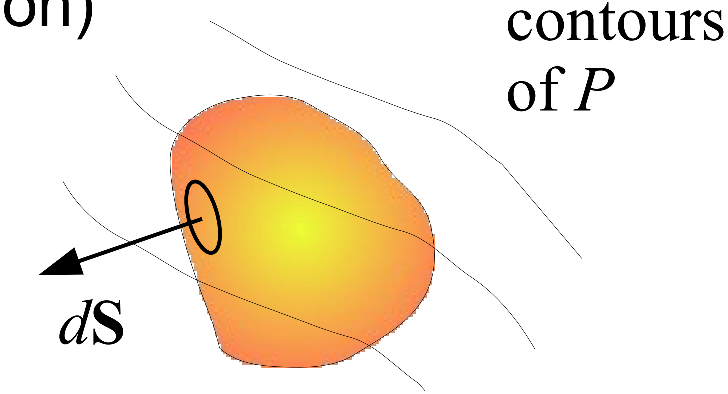
$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0}$$



$$dM = -\rho\, \mathbf{v}\, dt \cdot d\mathbf{S}$$

- Equation of motion (momentum equation)

$$\begin{aligned} \mathbf{F}_P &= \oint_{\partial V} d\mathbf{F}_P = -\oint_{\partial V} P\mathcal{I} \cdot d\mathbf{S} \\ &= -\int_V \nabla \cdot (P\mathcal{I}) dV \\ \Rightarrow \mathbf{f}_P &= -\nabla \cdot (P\mathcal{I}) = -\nabla P \end{aligned}$$



$$d\mathbf{F}_P = -P\, d\mathbf{S} = -P\mathcal{I} \cdot d\mathbf{S}$$

$$\mathbf{f}_\mathrm{L} = \frac{\sum_{\mathbf{r}_j \in V} q_j \mathbf{v}_j \times \mathbf{B}}{V} = \mathbf{J} \times \mathbf{B} \;\Rightarrow\; \boxed{\rho \frac{d\mathbf{v}}{dt} = -\nabla P + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g}} \qquad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

Fluid equations

- Conservation of mass and momentum equation:
  - four (scalar) equations
  - five fluid quantities: ρ, P, $\mathbf{v}$
  - more equations needed to close the set
- Ideal gas law

$$P = \frac{\mathcal{R}}{\mu}\rho T \; = n k_{\mathrm{B}} T$$

  - relates P to ρ, but introduces yet another quantity T
- Energy equation

$$\frac{\rho^\gamma}{\gamma - 1}\frac{d}{dt}\left(\frac{P}{\rho^\gamma}\right) = -L(T, \ldots)$$

γ is the adiabatic index

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

- Often, an equation of state, $P/\rho^\gamma = \mathrm{const.}$, used instead of IGL & EE

Ideal MHD Equations

- In the simplest form, the MHD equations can be written as

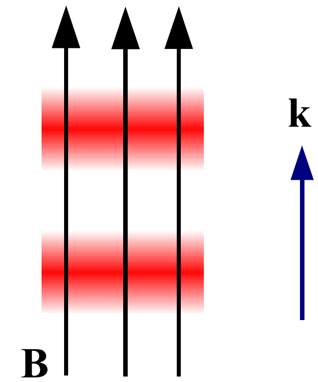
$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} &= -\nabla P + \frac{(\nabla \times \mathrm{B}) \times \mathrm{B}}{\mu_0} \\
\frac{d}{dt}\left(\frac{P}{\rho^\gamma}\right) &= 0 \\
\frac{\partial \mathrm{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathrm{B})
\end{aligned}
$$

- Closed set: eight (scalar) variables, eight (scalar) quantities

MHD waves

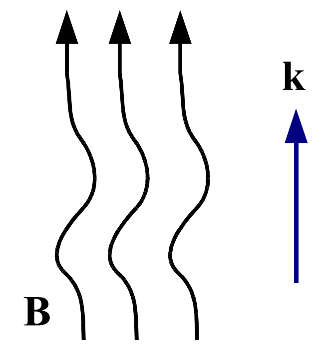
- “Small” perturbations propagate in form of MHD waves
- Signal speeds for principal MHD modes
  - longitudinal velocity perturbations propagating parallel to the ambient magnetic field (sound waves)

$$c_{\rm s} = \sqrt{\gamma p/\rho} = \sqrt{\gamma k_{\rm B} T/m}\,.$$



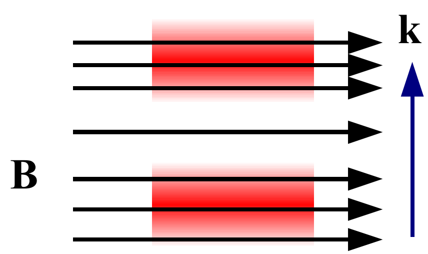
  - transverse velocity perturbations propagating parallel to the ambient magnetic field (Alfvén waves)

$$v_{\rm A} = \sqrt{\frac{B^2}{\mu_0 \rho}}$$



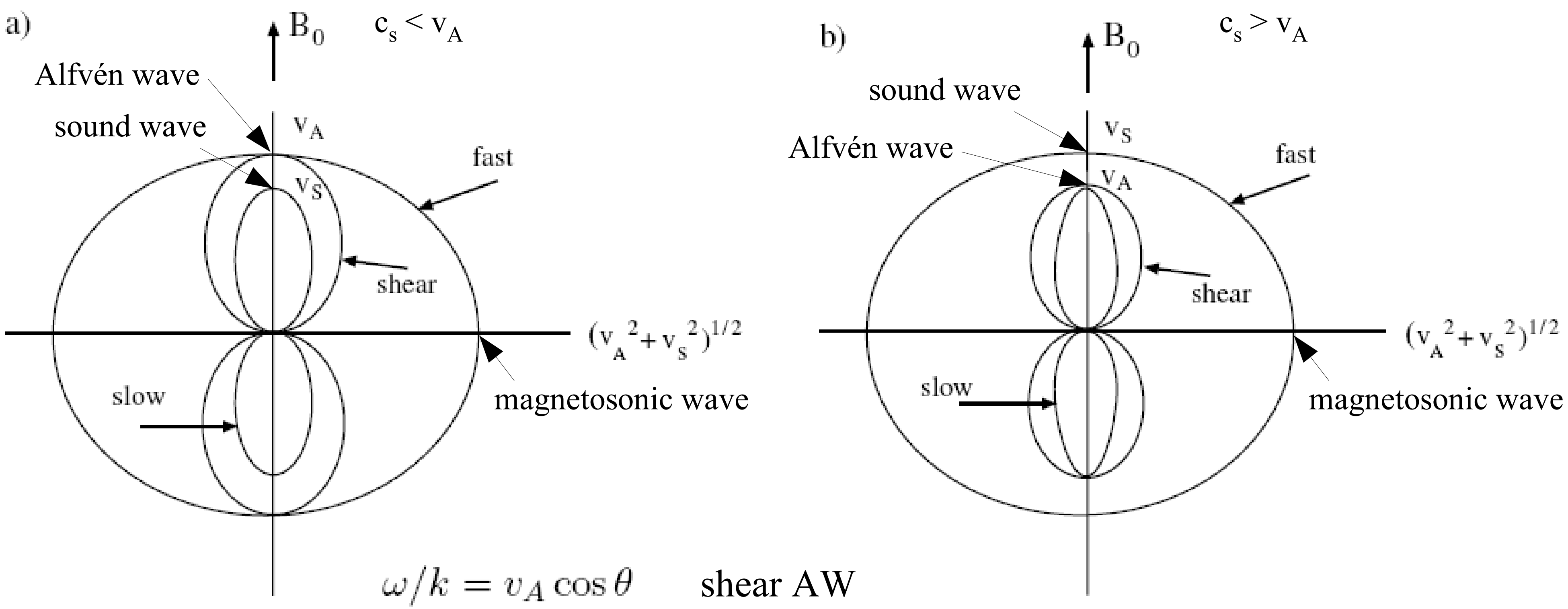
  - longitudinal velocity perturbations propagating perpendicular to the magnetic field (magnetosonic waves)

$$v_{\rm ms} = \sqrt{c_{\rm s}^2 + v_{\rm A}^2}$$



MHD waves at arbitrary angles

Wave propagation at arbitrary angles occurs in three MHD wave modes:
– slow MHD wave
– shear Alfvén wave (or intermediate MHD wave)
– fast MHD wave



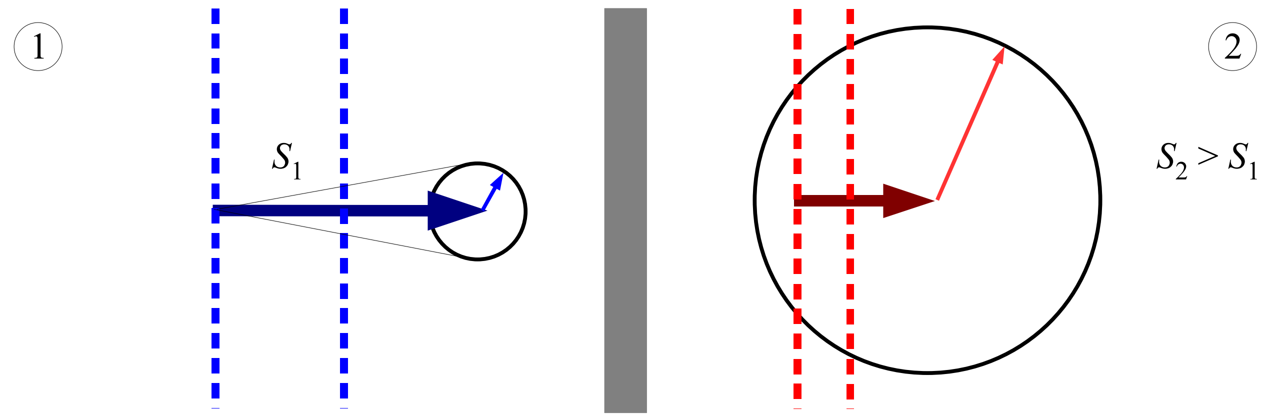
$$\omega/k = v_A \cos\theta$$ shear AW

$$\left(\frac{\omega}{k}\right)^2 = \frac{1}{2}(v_s^2 + v_A^2) \pm \frac{1}{2}[(v_s^2 + v_A^2)^2 - 4v_s^2 v_A^2 \cos^2\theta]^{1/2}$$

fast (+) and slow (–) MHD waves

Shock waves

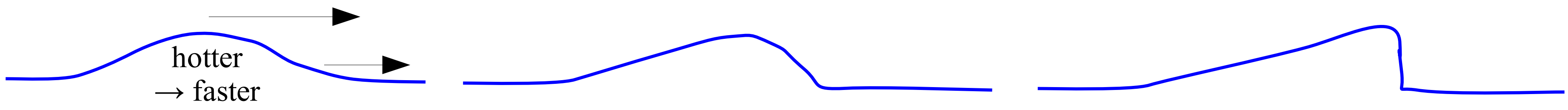
- **Definition**: Shock waves are thin transitions from supersonic to subsonic flow involving compression and dissipation



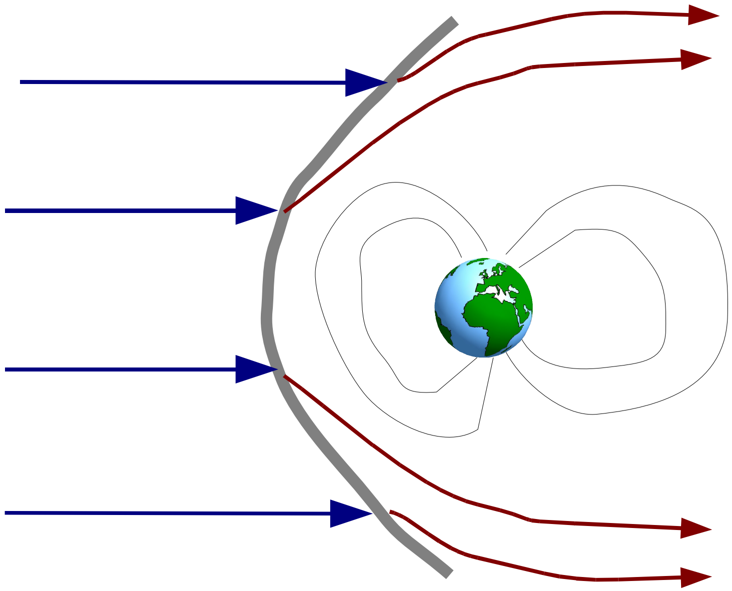
- **Note**:
  - **subsonic**: flow speed < signal speed (ω/k)
  - **supersonic**: flow speed > signal speed
  - **frame dependent** → above, flow speed measured in a coordinate system where the shock is stationary
  - regards flow speeds measured **normal to the shock front**

Different types of shocks

- Non-linear steepening of a large-amplitude sound wave:

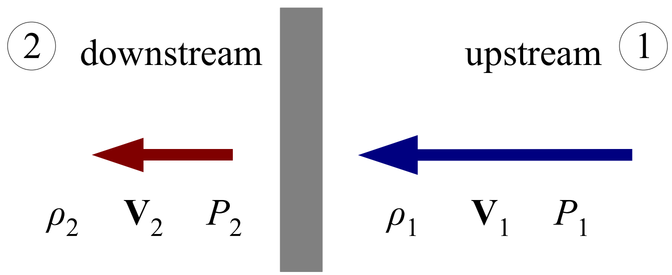


- Formation of a bow shock
  - needed for transmitting the information of an upcoming obstacle
  - upstream of a planetary obstacle
  - ahead of a CME



Hydrodynamic shocks

- Shock physics developed in the frame where the shock is stationary
  - assume: planar discontinuity
  - choose $\mathbf{v}_1 \parallel \mathbf{n}$ (always possible for shocks)



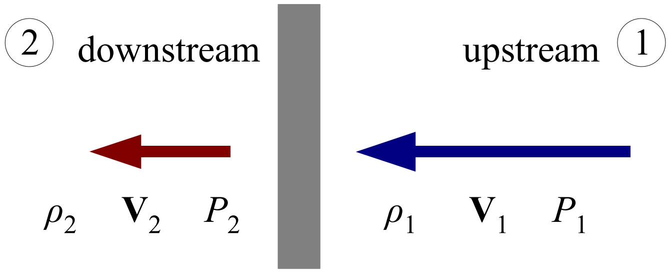
- Denote
  - upstream values by subscript 1
  - downstream values by subscript 2
- Conservation of mass, momentum and energy
  → jump conditions

$$\begin{aligned} \rho_2 v_2 &= \rho_1 v_1 && \text{Mass flux} \\ P_2 + \rho_2 v_2^2 &= P_1 + \rho_1 v_1^2 && \text{Momentum flux} \\ P_2 v_2 + (\rho_2 U_2 + \frac{1}{2}\rho_2 v_2^2) v_2 &= P_1 v_1 + (\rho_1 U_1 + \frac{1}{2}\rho_1 v_1^2) v_1 && \text{Energy flux} \end{aligned}$$

$U = P/(\gamma - 1)\rho$ Specific internal energy

- Solution to the jump conditions:

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$
$$\frac{v_2}{v_1} = \frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}$$
$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2-(\gamma-1)}{\gamma+1}$$

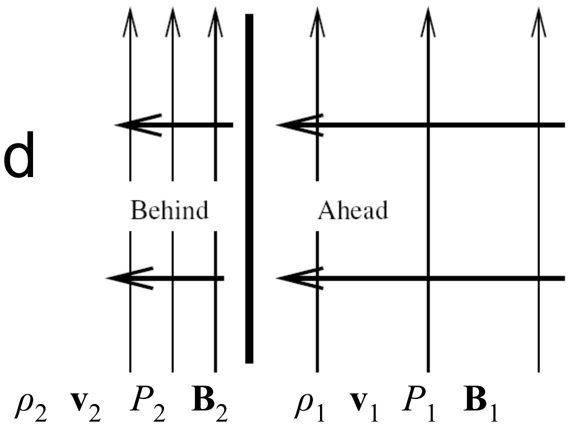


$M_1 = v_1/c_{s1}$ is the **sonic Mach** number

- $S = c_V \log P/\rho^\gamma$ must increase across the shock →

1. $M_1 \geq 1$, i.e., $v_1 \geq c_{s1}$ ahead the shock
2. $v_2 \leq c_{s2}$, flow is subsonic behind the shock
3. $P_2 \geq P_1$ and $\rho_2 \geq \rho_1$, the shock is compressive
4. $v_2 \leq v_1$ and $T_2 \geq T_1$, the flow is slowed down and the gas heated up
5. $1 \leq \rho_2/\rho_1 < (\gamma+1)/(\gamma-1)$, the maximum density ratio is $(\gamma+1)/(\gamma-1)$, but the pressure increases $\propto M_1^2$.

Perpendicular MHD shock



- Add a magnetic field perpendicular to the shock front; Faraday → $\mathbf{E}$ conserved
- Add the contribution of $\mathbf{B}$ to P and U →

$$\frac{v_2}{v_1} = \frac{\rho_1}{\rho_2} = \frac{1}{X}$$ just like in HD shock

$$\frac{B_2}{B_1} = X$$ i.e., $\mathbf{B}$ compressed just as the mass density (frozen-in field)

$$\frac{P_2}{P_1} = \gamma M_1^2\left(1-\frac{1}{X}\right) - \frac{1-X^2}{\beta_1}$$

$$\beta = \frac{2\mu_0 P}{B^2}$$ Plasma beta

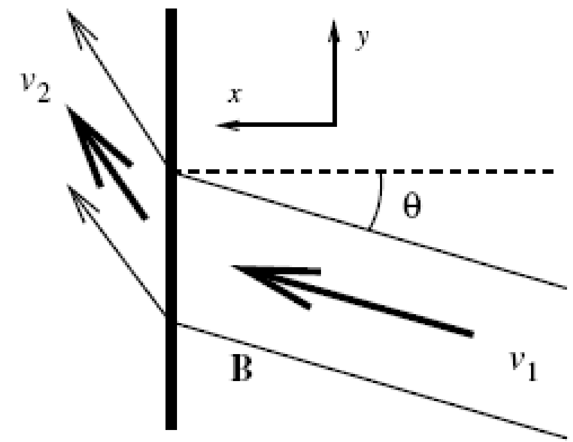
where $X = \rho_2/\rho_1$ is the positive root of

$$2(2-\gamma)X^2 + [2\beta_1 + (\gamma-1)\beta_1 M_1^2 + 2]\gamma X - \gamma(\gamma+1)\beta_1 M_1^2 = 0$$

- Note: $\mathbf{B}$ reduces the compression ratio below the HD value
  - upstream flow super-magnetosonic $v_1 > v_{\mathrm{ms}1} = \sqrt{c_{\mathrm{s}1}^2 + v_{\mathrm{A}1}^2}$
  - downstream flow sub-magnetosonic $v_2 < v_{\mathrm{ms}2} = \sqrt{c_{\mathrm{s}2}^2 + v_{\mathrm{A}2}^2}$

Oblique shocks

- Oblique shocks best treated in de Hoffmann–Teller frame
  - upstream field || flow
  - $\mathbf{E}$ vanishes $\rightarrow$ downstream field || flow
  - possible to choose, if $v_1/\cos\theta < c$
    if not, a frame where $\mathbf{n}\perp\mathbf{B}$ exists



- Jump conditions $\rightarrow$

$$\frac{v_{2x}}{v_{1x}} = \frac{\rho_1}{\rho_2} = \frac{1}{X}$$

$$\frac{v_{2y}}{v_{1y}} = \frac{v_1^2 - v_{\mathrm{A1}}^2}{v_1^2 - X v_{\mathrm{A1}}^2}$$

$$\frac{B_{2x}}{B_{1x}} = 1$$

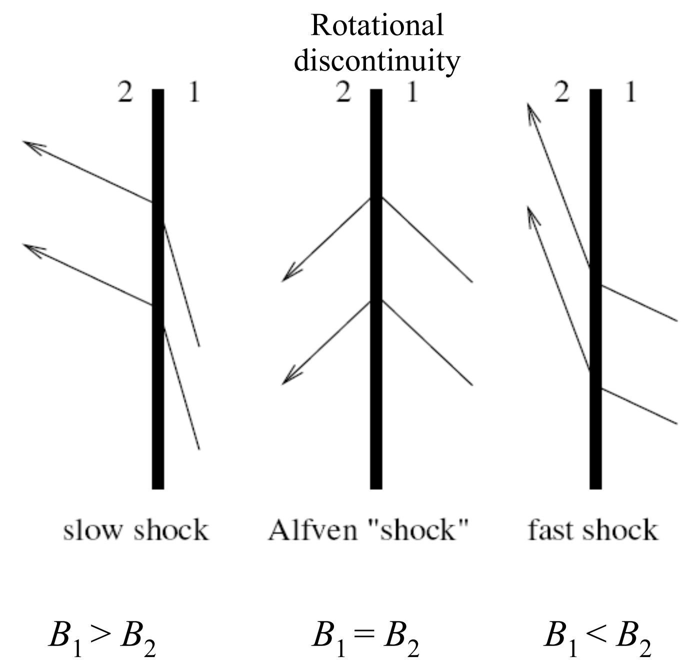
$$\frac{B_{2y}}{B_{1y}} = \frac{(v_1^2 - v_{\mathrm{A1}}^2)X}{v_1^2 - X v_{\mathrm{A1}}^2}$$

$$\frac{P_2}{P_1} = X + \frac{(\gamma-1)X v_1^2}{2c_{\mathrm{s1}}^2}\left(1 - \frac{v_2^2}{v_1^2}\right)$$

$$(v_1^2 - X v_{\mathrm{A1}}^2)^2\{X c_{\mathrm{s1}}^2 + \frac{1}{2} v_1^2 \cos^2\theta[X(\gamma-1) - (\gamma+1)]\} \quad (6.95)$$
$$+\frac{1}{2} v_{\mathrm{A1}}^2 v_1^2 \sin^2\theta \, X\{[\gamma + X(2-\gamma)]v_1^2 - X v_{\mathrm{A1}}^2[(\gamma+1) - X(\gamma-1)]\} = 0\,.$$

– Three roots:
- fast shocks
- slow shocks
- intermediate shocks a.k.a. rotational discontinuities
  $\rightarrow$ no compression, not a shock

- Magnetic field in oblique shocks

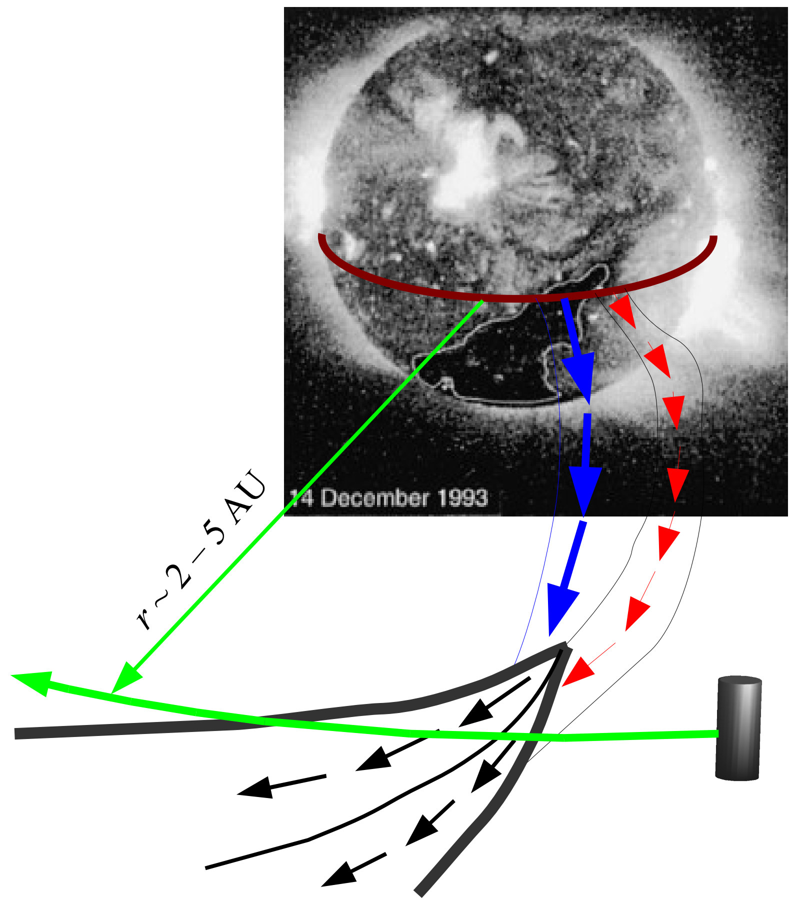


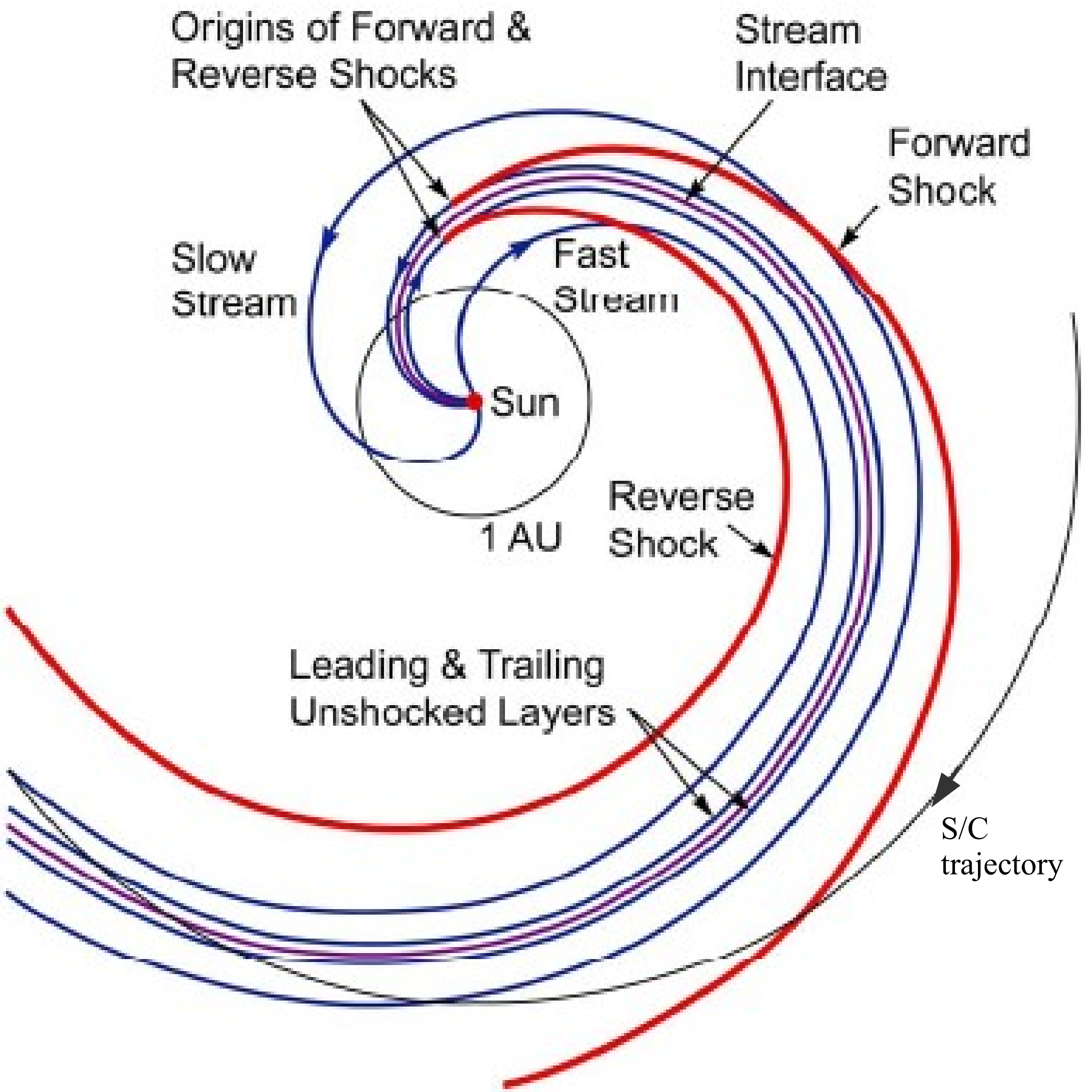
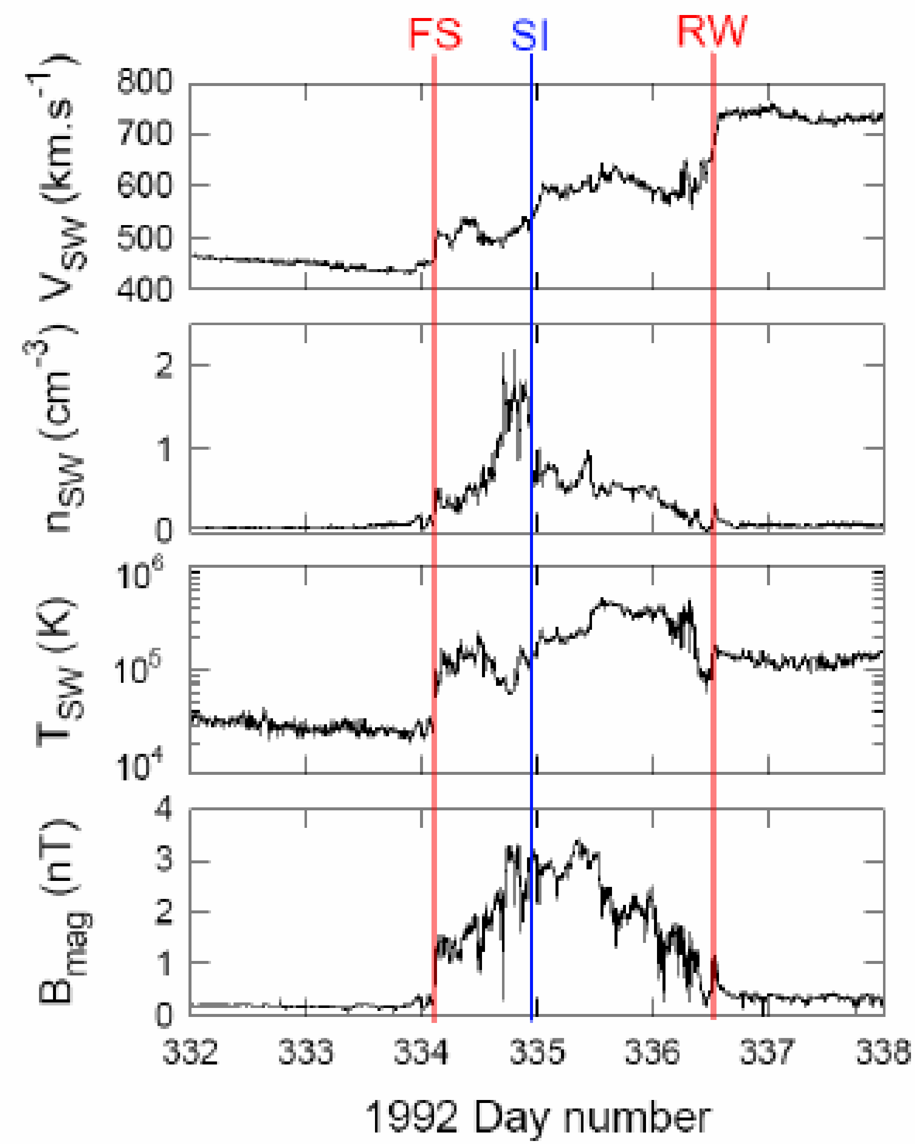
- Properties of fast and slow shocks:

1. They are compressive
2. B_x remains unchanged over the shock
3. They conserve the sign of B_y
4. At the slow shock $B_2 < B_1$
5. At the fast shock $B_2 > B_1$
6. v_{1x} exceeds the slow/fast speed ahead the shock while v_{2x} is smaller than the slow/fast speed behind the shock
7. $v_{2x} < v_{1x}$
8. At the limit $B_x \to 0$, the fast shock becomes perpendicular shock whereas the slow shock becomes a tangential discontinuity ($v_x \to 0$) with arbitrary jumps in v_y and B_y subject to total pressure balance over the shock.

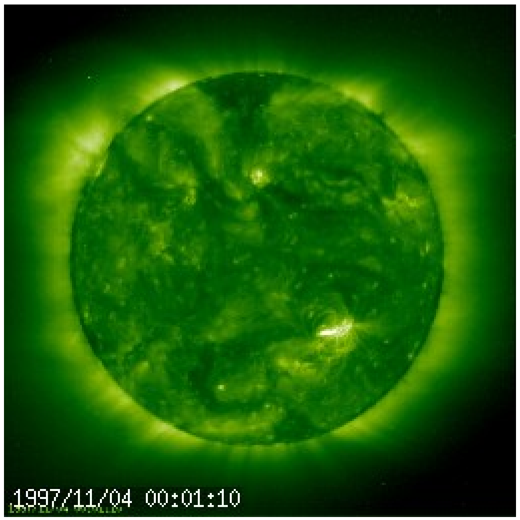
Corotating interaction regions

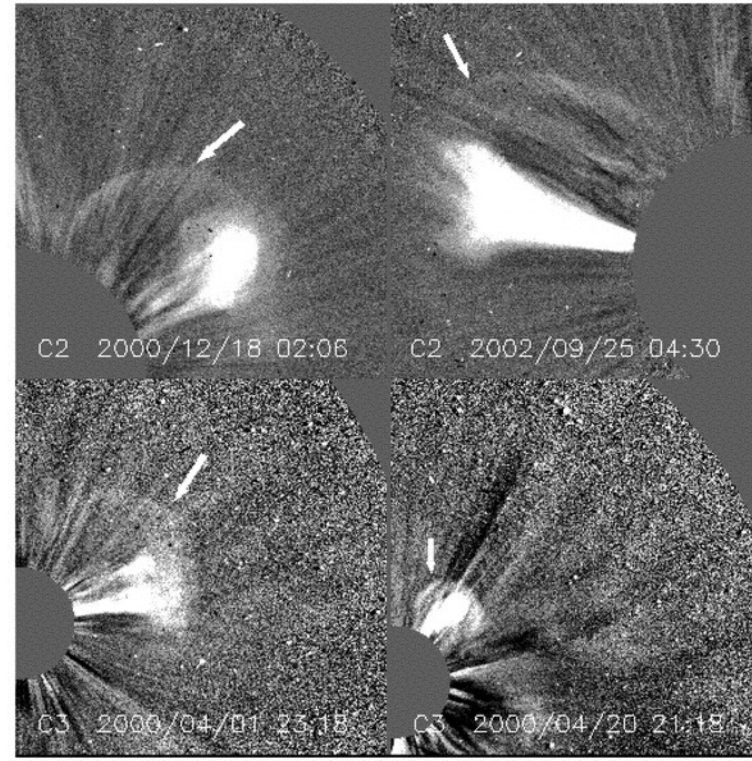
- Coronal hole boundaries are not aligned with latitudes
  - slow solar wind is caught up by a trailing faster stream
  - two shock waves (forward and reverse) form
  - the region between the shocks called a **corotating interation region (CIR)**
  - structure is stable for many solar rotations in the frame co-rotating with the Sun
- CIRs form at radial distances between 2 – 5 AU from the Sun

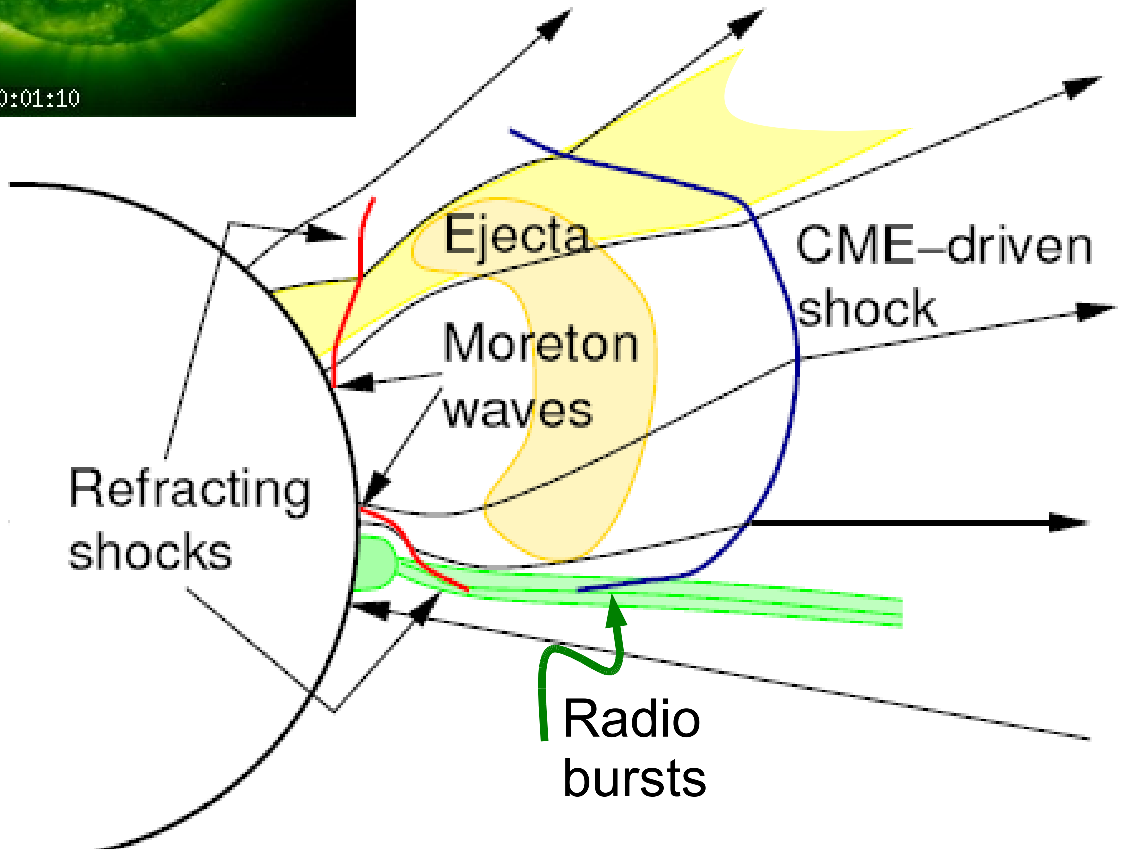


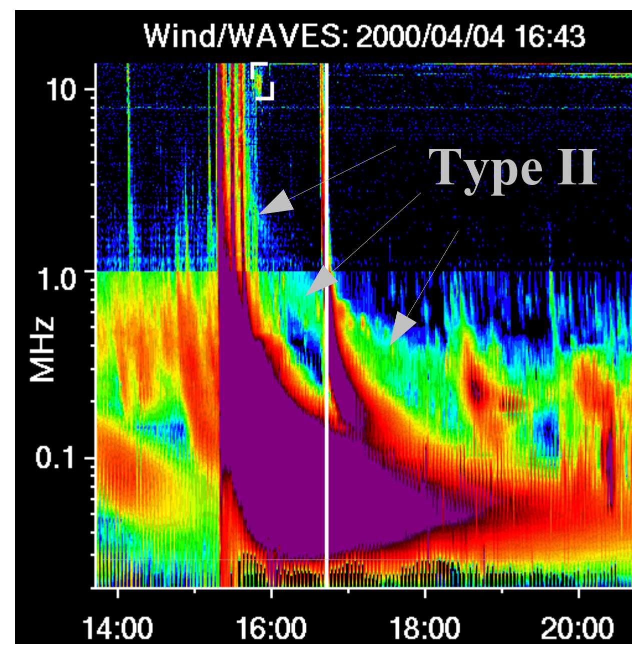

Origins of Forward &
Reverse Shocks
Stream
Interface
Forward
Shock
Slow
Stream
Fast
Stream
Sun
1 AU
Reverse
Shock
Leading & Trailing
Unshocked Layers
S/C
trajectory

FS
SI
RW
V_{SW} (km.s^{-1})
800
700
600
500
400
n_{SW} (cm^{-3})
2
1
0
T_{SW} (K)
10^6
10^5
10^4
B_{mag} (nT)
4
3
2
1
0
332
333
334
335
336
337
338
1992 Day number

Coronal shocks associated to eruptions





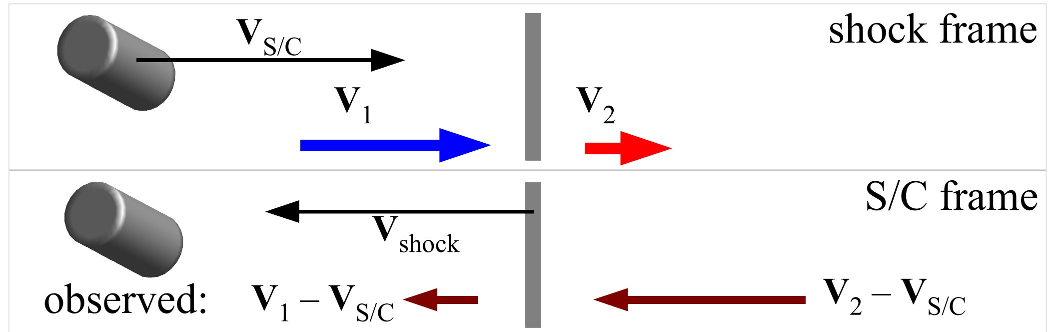


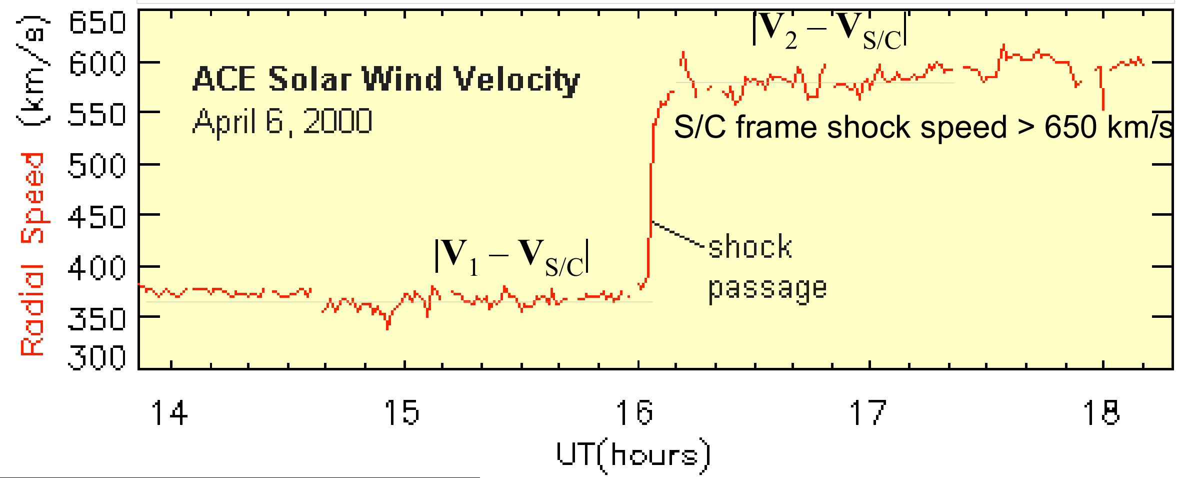


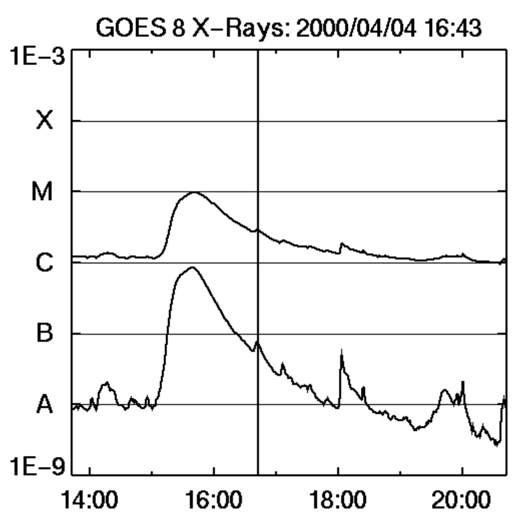
$$f_p(\mathrm{kHz}) = 9\sqrt{n_e(\mathrm{cm}^{-3})}$$

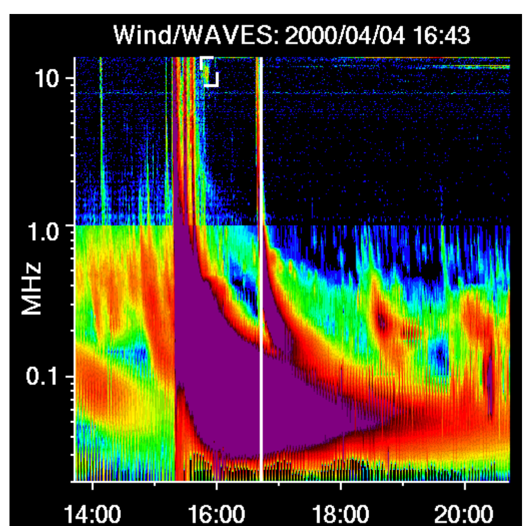
CME-driven interplanetary shocks

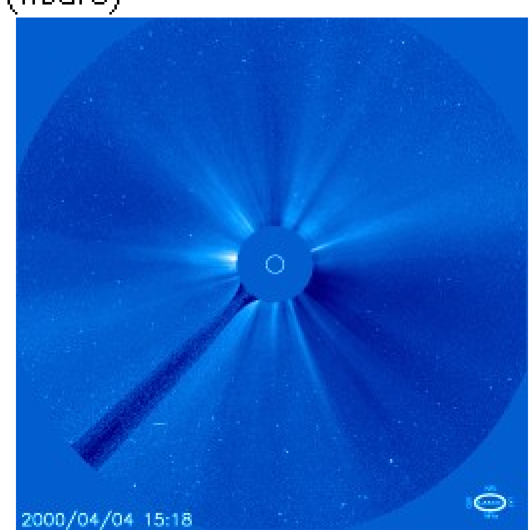
- CMEs super-magnetosonic → drive fast-mode shock waves
  - compress the magnetic field
  - amplify turbulence
  - **accelerate particles**



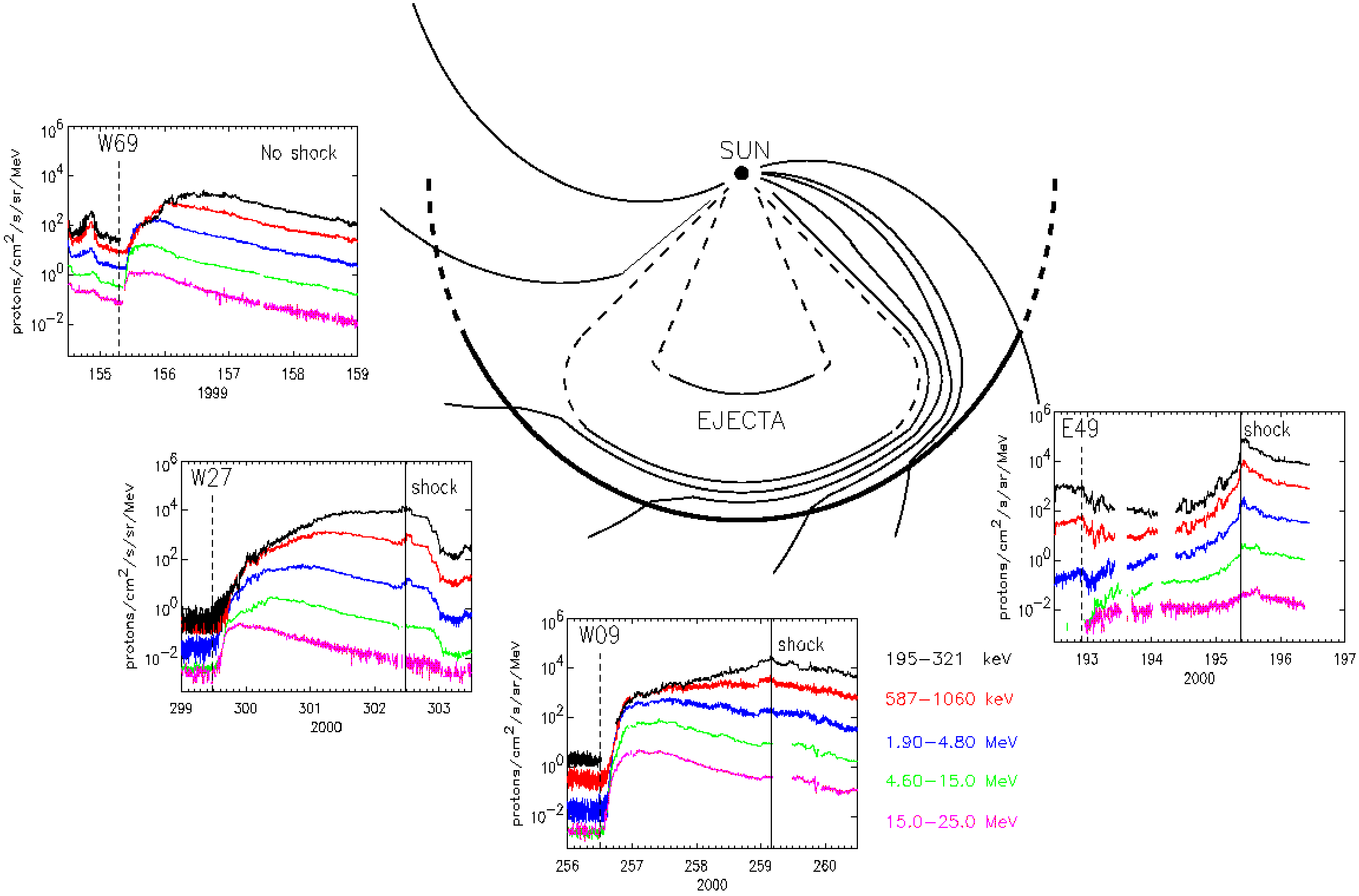








Longitude distribution of solar proton events

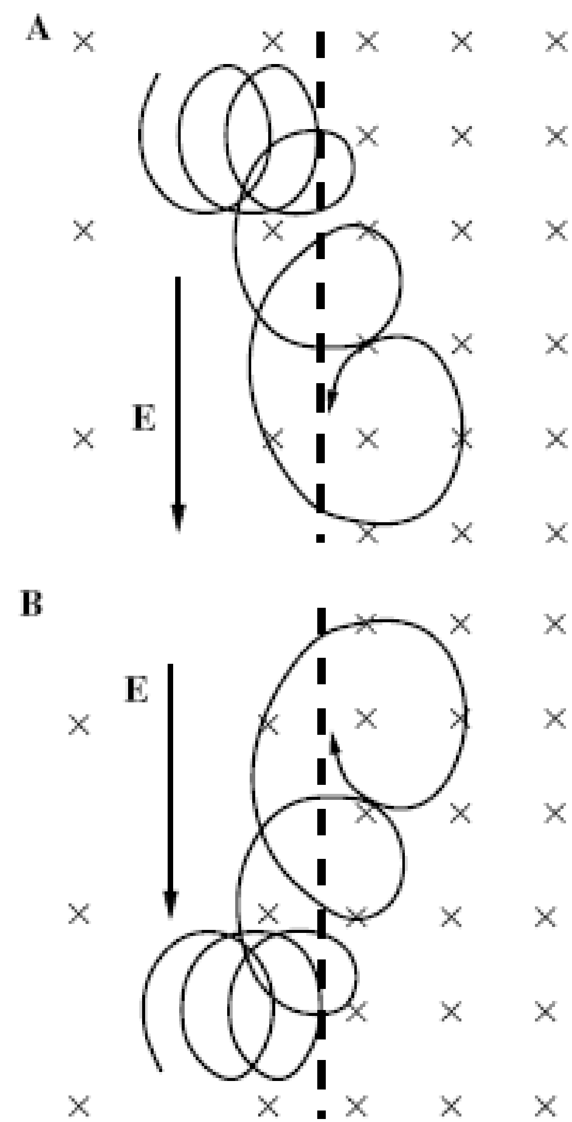


Shock drift acceleration (SDA)

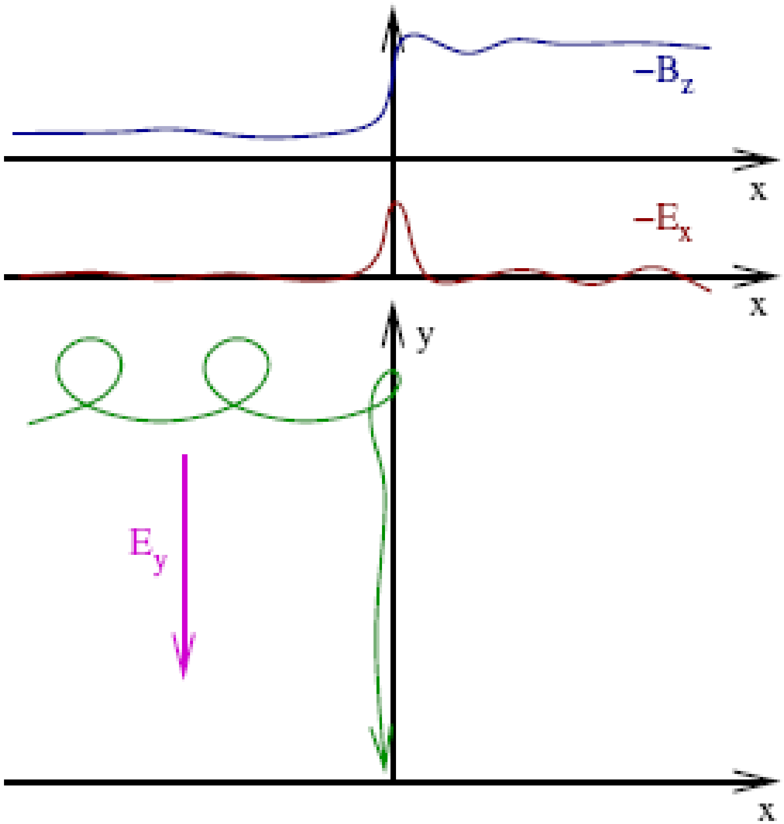
- Quasi-perpendicular shock
  - shock-frame electric field

$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} = -\nabla \phi$$

  - ions and electrons E×B-drift from the upstream region to the shock at velocity $\boldsymbol{V}$
  - magnetic field increased across the shock $\rightarrow$ **gradient drift**
    - ions drift parallel to the E-field $(d\phi / dt < 0)$
    - electrons drift anti-parallel to the E-field $(d\phi / dt > 0)$
    - $W = T + U = T + q\phi = \text{const.} \rightarrow$ both ions and electrons are accelerated
- Operates in oblique shocks as well.



Shock surfing



- $E_x = -d\phi/dx \Rightarrow$ Particles reflected
- Turned back by upstream B_z
- Eqs. of motion

$$\begin{aligned} \dot{p}_x &= q(E_x + v_y B_z) \\ \dot{p}_y &= q(E_y - v_x B_z) \end{aligned}$$

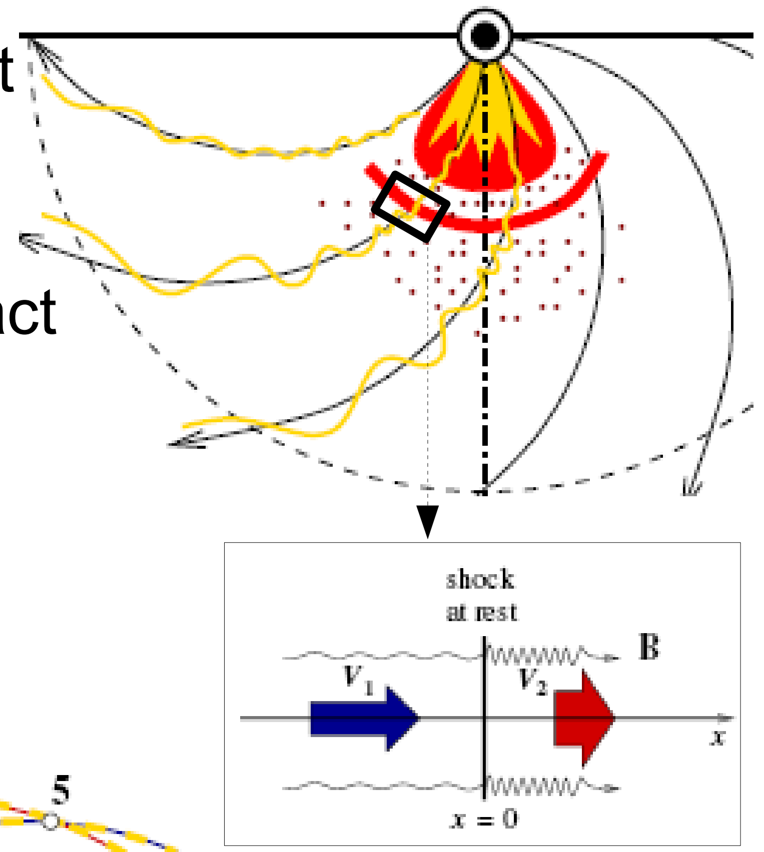
- Ass. v_x small
  $\Rightarrow$ Acceleration along $-y$
- particles trapped until

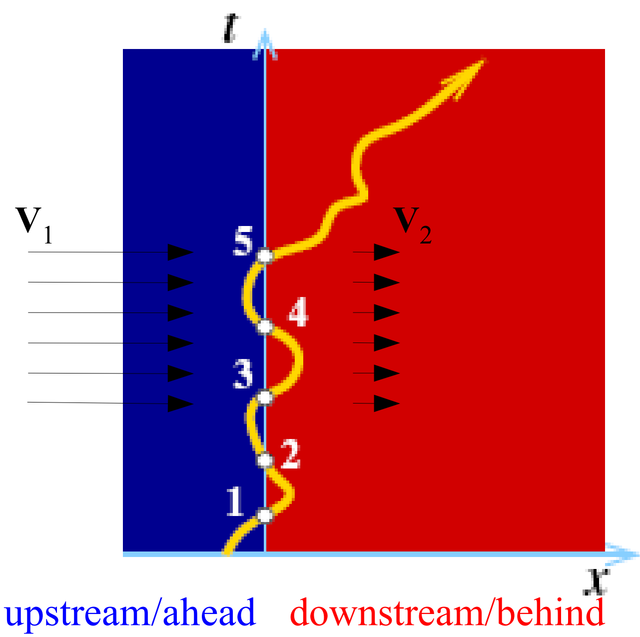
$$v_y \sim -\frac{E_x}{B_z} \ll -\frac{m_i^{1/2}}{m_e^{1/2}} M_A\, V_1 \quad \Rightarrow \quad \text{MeV energies (injector?)}$$

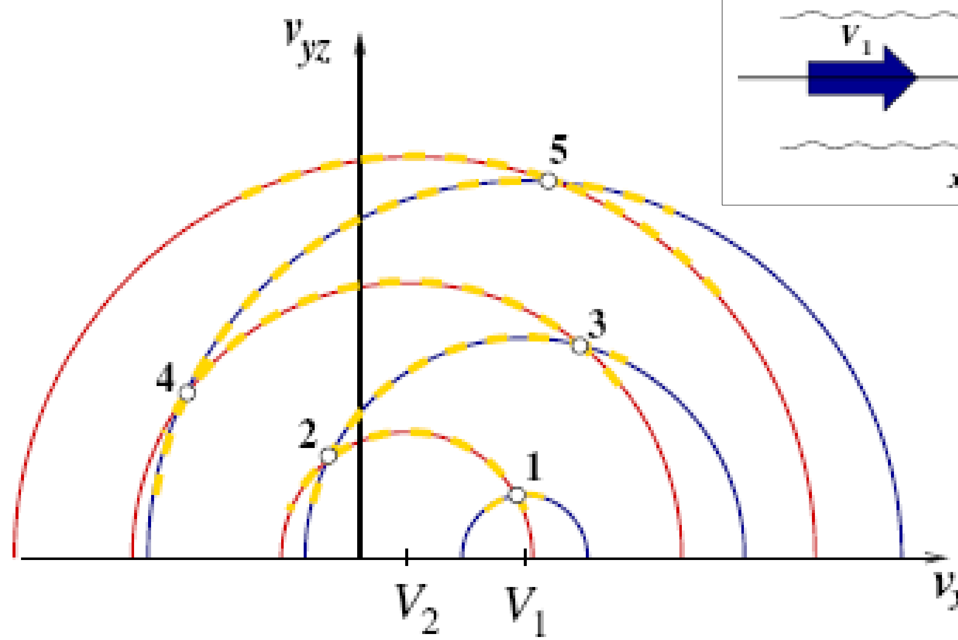
But: surfing requires an almost exactly perpendicular shock

Diffusive Shock Acceleration (DSA)

- One interaction with the shock does not provide very energetic particles → many interactions needed
- Particle scattering → particle can interact with the shock many times
- Simplest to describe for quasi-parallel shock waves







Summary

- MHD can describe plasma motion at macroscopic scales
- MHD supports three basic wave modes:
  - Alfvén wave (non-compressive)
  - Slow and fast MHD wave (compressive)
- MHD shocks are divided in
  - Fast shocks ($B_2 > B_1$)
  - Slow shocks ($B_2 < B_1$)
- Solar system contains many shocks
  - CMEs, CIRs, etc.
- Shocks accelerate particles to high energies
  - E.g., SEPs (but actually galactic cosmic rays as well!)
  - Mechanisms: shock-drift, shock surfing, and diffusive shock acceleration