RADIO BRIGHTNESS DISTRIBUTION OF THE SUN AT A WAVE-LENGTH OF 10.7 CENTIMETRES JUNE 30, 1954

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Abstract

A partial eclipse of the sun was observed on June 30, 1954 at the Radio Observatory of the National Research Council in Ottawa. During the eclipse, a record of the varying ten-centimetre solar radiation from a 2-degree region containing the sun and moon was obtained with a paraboloid reflector ten feet in diameter. After the eclipse, when the sun drifted through the receiving pattern of a 150-foot wave-guide array, solar radio emission was obtained in north-south strips approximately 1/8-degree wide. These observations have been interpreted in terms of a disk with a radius 1.14 times that of the photosphere and with the equatorial regions brighter than the polar regions. Equivalent temperature at the centre of the radio disk is about 33,000°K.

The existence of radio waves originating from the sun has been known for a little more than a decade and has both supplemented and extended our physical knowledge of the solar atmosphere. Many of the early observations were taken with instruments of such low resolving power that no angular discrimination was made between points on the solar disk and in consequence, the total emission from the disk was received. Only recently, with the construction of large antennas, has it been possible to isolate regions on the disk with an accuracy of a few minutes of arc. Eclipse observations offer a means whereby sources of radio waves may be located with great accuracy although an instrument of low resolving power is used. The positioning of an emitting source is limited to a single parameter since the source may lie anywhere on the circle representing the moon's edge. This ambiguity makes the interpretation of eclipse records difficult. The limiting accuracy with which a point may be located anywhere upon a circle in a radial direction is dependent upon the diffraction of the radio waves by the moon's limb. At a wave-length of 10.7 centimetres, this amounts to about 3.5 seconds of arc, an equivalent resolving power which can only be attained by an antenna of aperture of 4 miles. Unfortunately the full use of this equivalent resolving power is usually limited by uncertainties in the record introduced by the residual receiver noise.

The radio observations of the partial solar eclipse which occurred in Ottawa on June 30, 1954 were taken at the Radio Observatory of the National Research Council. The observations were started before the

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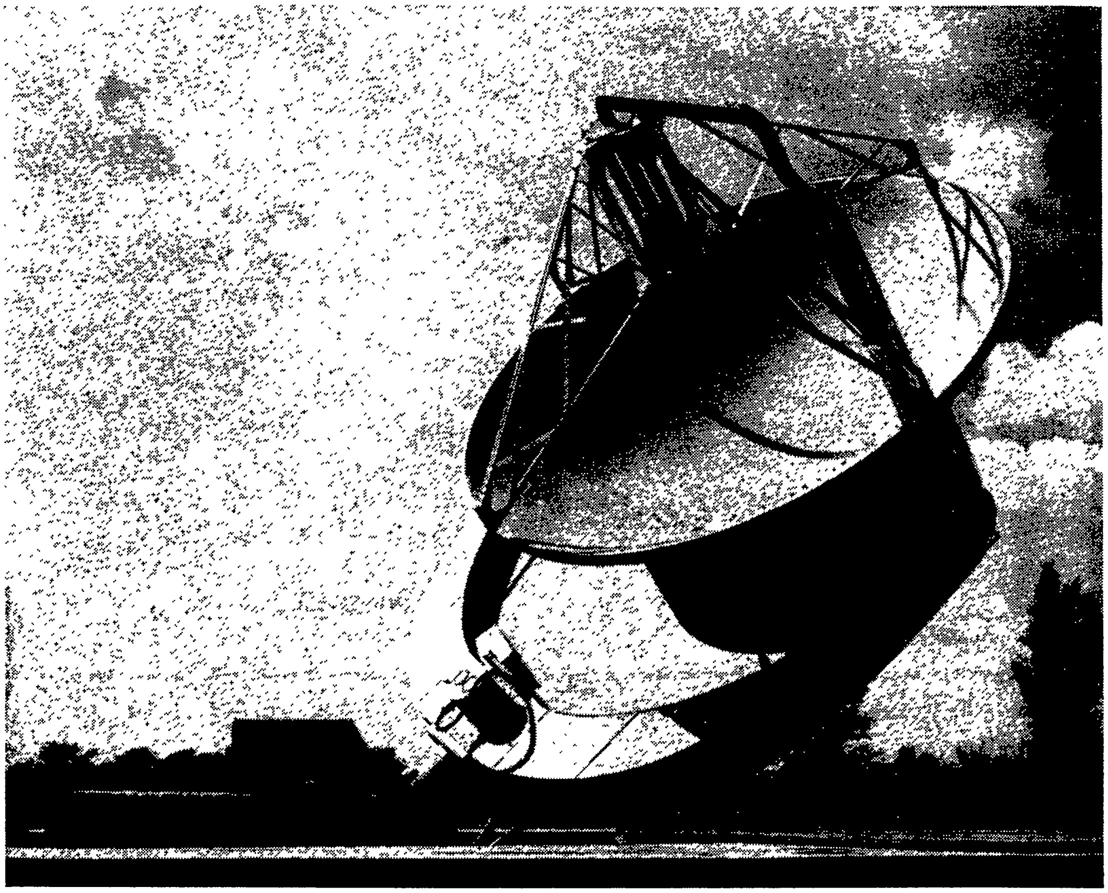

FIG. 1—10.7-centimetre radio telescope at National Research Council, Ottawa, Canada—10-foot diameter paraboloid.

first contact and continued until the eclipse was well over. As the moon covered various portions of the sun's disk the total solar radio output was detected and recorded. Two such records, or eclipse curves, were obtained: one with the recently installed 10-foot parabolic reflector (figure 1), which has a receiving pattern of 2.5 degrees to the half-power points, and the other with the 4-foot parabolic reflector, which has a receiving pattern of 7 degrees to the half-power points. Both antennas were motor-driven so that the acceptance cone remained on

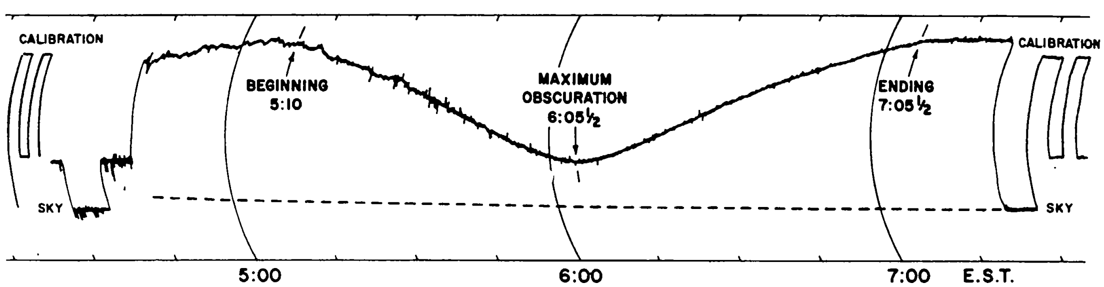

FIG. 2—Eclipse curve, June 30, 1954, obtained with 10-foot paraboloid.

the region containing the sun and moon. Since the larger instrument receives more solar energy than the smaller one, the eclipse record so obtained (figure 2) will have greater relative accuracy and consequently was used in the subsequent analysis. The absolute value of the solar flux was obtained with the smaller antenna since its various parameters have been well investigated.

Since the eclipse commenced shortly after sunrise and lasted until an elevation of 30 degrees had been reached, a correction for the atmospheric absorption and the emission from the sky and earth had to be determined. Patrol observations were taken on the days both preceding and following the eclipse. On each day, after the sun had traversed the path of interest, the antenna was driven back over the same path in order to determine the emission from the sky and earth. In figure 2 this is shown as a dashed line. If the assumption is made that the sun is a signal generator of constant output, then an appropriate correction factor may be found for each angle of elevation and can be used to restore the observed low angle solar emission to the value observed at a higher elevation. Examination of several days' records, including the day of the eclipse, showed some fluctuations just after sunrise. Since most of these were of a non-repeatable nature, it was thought that they were probably of atmospheric origin and not related to any pattern caused by the interference of direct solar rays and a reflection from the ground. This unexplainable roughness of the record during the early part of the eclipse introduced considerable error in comparison to the subsequent period, so that it was decided to use only the smoothest section of the record—a section commencing before maximum obscuration and extending to beyond last contact. For this period of interest the atmospheric and earth corrections are very small, varying from 2 per cent. to 0.25 per cent. In addition, during the first part of the eclipse there were a few minutes of circuit instability which cured itself before the time of maximum obscuration. This appears as sharp lines across the trace but otherwise does not spoil the record. The relative gain of the radiometer was measured by reference to the noise emitted by a fluorescent lamp both before and after the eclipse. These measurements showed that during the three-hour period of observations the gain of the radiometer had changed by only 1 per cent.

The second set of radio observations was taken about five hours after the eclipse when the sun was near the zenith and passing through the narrow fan-shaped receiving pattern of the long array (figure 3). This antenna is 150 feet long and has a receiving pattern approximately 1/8-degree east-west and 20 degrees north-south. The output of the

FIG. 3—10.7-centimetre radio telescope at National Research Council, Ottawa, Canada—150-foot slotted waveguide array.

receiver when the sun drifts through the narrow section of the antenna pattern has been called a drift curve (figure 4) and its analysis gives some knowledge of the angular distribution of the radio emission from the solar surface (Covington and Broten, 1954). Four drift curves which differ from each other only by amounts that may be ascribed to the receiver fluctuations were taken with this instrument. After normalizing each with respect to the 4-foot reflector, the average of the four curves was obtained for use in the subsequent analysis. This average curve is very symmetrical, the area of one half agreeing with that of the other to one part in 250, and is evidence that the radio sun is axially symmetrical. The patrol days before and after the day of the eclipse showed similar drift curves, from which it is concluded that there were no radio emissive regions present on the visible hemisphere during this period. The two halves of the average drift curve were again averaged to give the drift curve readings that were analysed. It is interesting to note that this curve is very similar to that of the quiet sun lower envelope found in 1952 (Covington and Broten, 1954).

Radius of the Radio Sun

The radius of the radio sun is an important characteristic and is perhaps best determined, to a first approximation, from the eclipse curve rather than from the drift curve. The shadow of the moon's edge is very sharp in comparison with the fan-shaped antenna pattern of the long array, so that the edge of the radio disk is located by the transition

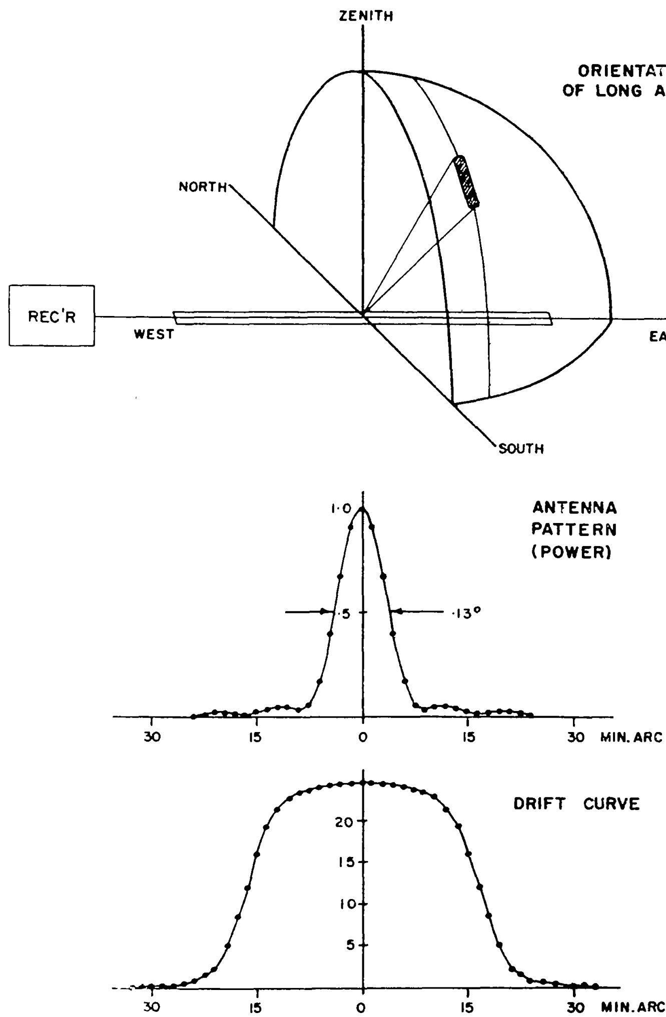

FIG. 4—Drift curve, June 30, 1954, obtained with 150-foot waveguide array: orientation of the array with antenna pattern hatched; antenna pattern in east-west direction.

between the eclipse curve and the straight line representing the total solar emission. In the absence of any precise knowledge on the variation of the radio brightness with radial distance, simple geometric criteria were used to determine the time of radio contact. A straight line was drawn through the section of the eclipse curve near the end and was projected until it intersected the straight line representing the total emission. The point of intersection of these two lines thus determines a minimum value for the radius of the radio sun. To determine a maximum radius, the eclipse curve was hand-smoothed and the point noted at which it first deviated from the straight line representing the full value. In the model of the radio sun used to analyse the shape of the curves, the maximum radius is used to ensure that the solar field is sufficiently large to include all of the solar emission. The two values of the radius are given in Table I together with other data for the optical and radio eclipses.

TABLE I

CIRCUMSTANCES OF OPTICAL AND RADIO PARTIAL ECLIPSE OF THE SUN AT OTTAWA, CANADA, JUNE 30, 1954

		Optical	Radio
Circumstances of the Eclipse			
First contact		$10^h9^m56.8^s$	....
Middle:	*Calculated*	$11^h5^m23.9^s$	
	Observed	$11^h5^m23.5^{*s}$	$11^h5^m24^s$
Last contact		$12^h5^m36.7^s$	$12^h7^m22^s$ (earliest) $12^h9^m45^s$ (latest)
Magnitude		0.857	
Solar Radius			
Optical	15′43.9″	(1.00 $R_\odot$)	
Radio	16′41″ (min.)	(1.06 $R_\odot$)	
	18′ 0″ (max.)	(1.14 $R_\odot$)	

*Courtesy J. Locke, Dominion Observatory, Ottawa.

Analysis of Curves

The radio waves emitted from the sun induce random voltages in the antenna such that the radiation resistance of the antenna assumes an equivalent temperature T_{ant} given by the equation,

$$T_{\text{ant}} = \frac{G_0}{4\pi}\int g(\theta, \phi)\, t\,(\theta, \phi\,)d\,\Omega,$$

where $g(\theta, \phi)$ is the normalized receiving pattern, G_0 is the maximum antenna gain, $t(\theta, \phi)$ is the equivalent temperature in the direction (θ, ϕ), $d\,\Omega$ is an element of solid angle.

This equivalent temperature of the antenna is measured in terms of the current output of the radiometer and ultimately is given a magnitude by reference to the known thermal radio emission from a resistor substituted for the antenna. Since both G_0 and $g(\theta, \phi)$ are known from engineering practice, the solution of the integral equation provides certain information about the radio emission as given by an equivalent temperature in various directions of space.

In applying this integral equation to analyse the results of the two experiments, certain instrumental conditions are of importance and modify the form of the equation. In the case of the eclipse curve, the broadness of the receiving pattern in comparison with the sun permits $g(\theta, \phi)$ to be set equal to one. Consequently at any given time corresponding to a given point on the eclipse curve, the magnitude of the radio emission is related to a particular sun-moon configuration, each element of area on the unobscured solar disk emitting radio waves with a certain intensity. In the case of the drift curve, the angular width of the receiving pattern in one dimension is smaller than the angular size of the solar disk so that the exact form of $g(\theta, \phi)$ in that direction must be used. Thus at any given time or point on the drift curve, the magnitude of the radio emission is proportional to the intensity of the radio waves emitted by north-south strips on the sun each weighted by the gain factor which is a function of angle in the east-west direction.

Since the unknown solar temperature distribution requires two space variables for its description, while either of the eclipse or drift curves is described in terms of a single time parameter, a solution to the modified integral equations may only be obtained after an assumption is made about the symmetry of the unknown solar temperature. This assumption effectively replaces the two unknown variables by a single variable which is now related to the single time parameter of either the eclipse or drift curve. The unknown distribution was assumed first to have circular symmetry about the centre of the disk, and separate solutions were found for both the eclipse and the drift curve. These two solutions differed from each other. This seemed unreasonable since the monitoring of the total solar emission showed that there was no change in the total emission within the five hours separating the eclipse and the drift of the sun through the long array pattern. As a further check of this incompatibility, a drift curve was constructed using the temperature distribution derived from the eclipse curve and compared with the observed drift curve; and an eclipse curve, constructed by using the temperatures derived from the drift curve, was compared with the observed eclipse curve. In both cases there was again a decided lack of agreement which was taken to indicate that the assumption of circular symmetry was not valid. Other indications of an asymmetrical radio sun based upon work at other institutions have just recently appeared (Swarup, G. et al., 1955).

In order to remove these discrepancies, it was necessary to make a more complicated assumption about the temperature distribution on the solar disk—namely that the temperature of the equatorial regions could be different from that of the polar regions. A model with this possibility is shown in figure 5 and was used in the subsequent analysis. This geometrical model defines a region of radio emission that is circular with a radius of 18 minutes of arc and centred upon the optical sun. It is subdivided into a central disk of 12 minutes of arc radius, surrounded by four rings each 1.5 minutes of arc wide. The central disk was not subdivided since previous work with the long array had shown that this region of the sun did not contain much structure. The choice of the width of the outer rings was influenced by the fact that all previous work on the drift curves had been undertaken with such an interval. The rings are further divided into two groups: one comprising a polar region, the other an equatorial region. The radii forming the boundaries between these groups are at an angle of 38 degrees with a line parallel to the moon's path. The axis of the radio model is rotated about 3 degrees from the axis of the sun but this effect was considered of small importance in the analysis of the curves. Although this model was designed to provide for asymmetrical temperature distributions on the solar disk, it includes the symmetrical model as a special case and if the analysis of the curves had required a circularly symmetrical distribution as a solution, it still could have been obtained.

After the unknown temperature distribution was assumed to have the asymmetry depicted by the geometrical model, the drift and eclipse

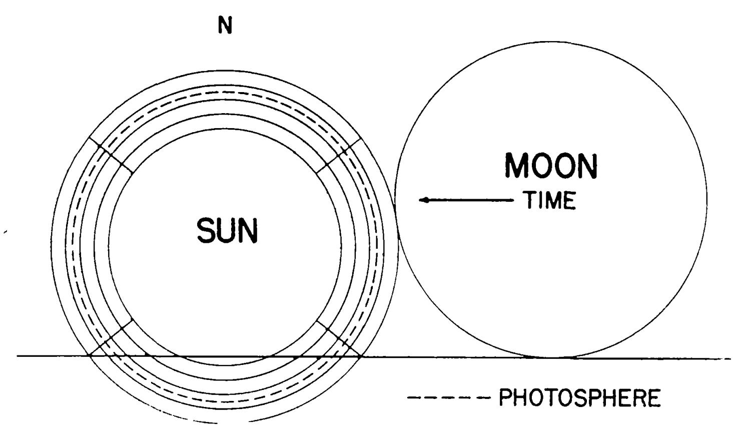

FIG. 5—Solar field used in the analysis of the curves and sun-moon configuration for the Ottawa eclipse.

curves were separately analysed in a manner that will be described subsequently. It was found that each of the original curves gave rise to temperature distributions which had an equatorial region that was brighter than the polar region. Since solutions which satisfied one of the original curves to within the expected accuracy of observation also satisfied the other independent curve to an accuracy of two or three times the expected error of observation, it was concluded that the model was satisfactory, that no significant improvement would be made by alterations such as changing the angle of the dividing line between the polar and equatorial regions, and further, that the best results could be obtained by solving the two curves simultaneously. The method of obtaining the simultaneous solution is the same as that used in examining either curve alone and will now be outlined.

A general method for the solution of an integral equation in the presence of errors involves the variation of the unknown distribution so that the observed curves may be reconstructed within the errors of observation. In the model the radio emission occurs uniformly from within the various cells and the integral equation may be replaced by a set of linear equations.

At any particular time the intensity of the radio emission as received at the antenna terminal of either the 10-foot reflector or the long array may be expressed as a sum of terms:

$$T_j = \sum^{i} a_{ij} t_i \, ,$$

where T_j is the reading of either the eclipse or drift curve at the time "j", t_i is the unknown temperature in the "i" cell and is the same for both the eclipse and drift curves, a_{ij} is proportional in the case of the eclipse curve to the exposed area of the "i" cell, and in the case of the drift curve to the summation of the product of the cell areas and the appropriate antenna gain. Each observed curve is thus described by an appropriate set of linear equations, each equation corresponding to a point on the curve. In the absence of any errors, any set of "i" equations would be sufficient to solve for the t_i. However, there are errors of various kinds associated with the observed curves so that incompatible solutions will be obtained from different sets of "i" equations. For example, the reading errors may be such that a set of equations based upon observations near the end of the observed curves would give a solution incompatible with that obtained from a set of equations based on observations taken at the middle of the curves. In this situation the method of least squares must be applied and a solution obtained from the normal equations which are derived from all the original equations. Thus the original "*j*" equations are replaced by "*i*" normal equations of "*i*" variables and the solution of this new set is "best" in the sense that the mean square error between the observed curve and the one calculated from the solution is a minimum.

The values of temperature for the adjacent cells or rings of the model, as derived from the normal equations, often show large fluctuations. These will have no physical meaning and are certainly unreal when negative temperatures are obtained. Such solutions arise from fitting the observations with an accuracy greater than that justified by the experimental situation. In order to avoid solutions with too much detail of a fictitious nature, a modified method of analysis incorporating smoothing of the unknown temperature distribution was developed. This smoothing is introduced by adding constraints to the normal equations which confine the unknown temperature variables to polynominal forms of low degree. The direct addition of constraints through the use of undetermined multipliers proves difficult since the greater the amount of smoothing introduced, the greater are the number of constraints and the larger the determinants that enter into the solution. A more convenient method of working directly with the coefficients of the various polynominals was devised by a suitable transformation and enabled a series of solar temperature distributions of known and increasing complexity to be found. The choice of a solution is made after all the fitting errors have been computed.

The minimum fitting error for any one solution is defined as the sum of the squares of the differences between the observed readings and the readings calculated from that solution. It must be compared with the estimated error known to be associated with the equipment. The solution which has a fitting error comparable to this estimated error gives the best interpretation of the curve; those with a greater fitting error are acceptable but will be smoothed too much, while those for which the fitting error is less most likely will contain fictitious variations.

The first in the series of temperature distributions to be used in the analysis contains the least amount of detailed information in the sense that all parts of the radio disk are assumed to emit uniformly. The solution is found—in this case a constant value of temperature. A curve is reconstructed and compared with the observed curve. If the fitting error associated with this type is greater than the estimated experimental error, more information is available and another type of temperature distribution with more complexity is assumed. This second type is based upon a linear variation of temperature between the various rings of the model. The solution is obtained and again the observed and calculated curves are compared. If there is still a significant difference between these curves, still more complexity as given by a parabolic variation between the temperature of the various rings may be introduced. When a fit comparable to the estimated error is found, this process may be discontinued since proceeding to types of greater complexity is likely to result in solutions which are influenced by accidental errors.

In the various types of temperature distribution based upon the model the successive variables introduced into the analysis allow the functional form of the temperature in the north-south direction to be independent of that for the temperatures in the east-west direction. The temperature of the central area is common to both the north-south and east-west variations.

Solutions Obtained from the Simultaneous Analysis of Eclipse and Drift Curves

The eclipse and drift curves were analysed simultaneously according to the method outlined and a series of solutions proceeding from simple to complex forms were obtained. These solutions are shown in Table II,

TABLE II
SIMULTANEOUS SOLUTIONS OF ECLIPSE AND DRIFT CURVES

Case	Type	Number of Parameters	M.S. Error (Eclipse)	M.S. Error (Drift Curve)	Temperature (degrees Kelvin) r=0′–12′	12′–13.5′NS 12′–13.5′EW	13.5′–15′NS 13.5′–15′EW	15′–16.5′NS 15′–16.5′EW	16.5′–18′NS 16.5′–18′EW
(a)	Uniform	1	36.78	52.16	32900	32900 32900	32900 32900	32900 32900	32900 32900
(b)	Uniform N–S Linear E–W	2	45.29	30.94	32100	32100 33400	32100 34600	32100 35800	32100 37100
(c)	Linear N–S Uniform E–W	2	10.32	46.18	35100	32500 35100	29900 35100	27200 35100	24600 35100
(d)	Linear N–S Linear E–W	3	14.91	31.22	34300	31800 35400	29300 36600	26700 37700	24200 38800
(e)	Linear N–S Parabolic E–W	4	10.92	5.05	32600	31100 50600	29500 53900	28000 42500	26400 16200
(f)	Parabolic N–S Linear E–W	4	7.71	15.32	37700	−2400 37000	−9200 36400	17300 35700	77100 35000
(g)	Parabolic N–S Parabolic E–W	5	10.03	5.08	33900	21000 48100	18000 50400	24900 40900	41800 19600
Estimated Experimental M.S. Error			(6.7)	(8.0)					

together with the corresponding mean-square fitting errors which must be compared with the mean-square errors based on the experimental situation. The experimental error for any point on the eclipse curve is 0.5 per cent., and for the system of points involved in the analysis this percentage gives an estimated total mean-square error of 6.7. For each point on the drift curve a relative accuracy of 1.5 per cent., was estimated, giving an estimated total mean-square error for the curve of 8.0. With these values of error in mind, an examination of the table shows that solutions (e), (f) and (g) provide the best approximation to the observed results. Of these, (f) may be discarded immediately, since with its negative temperatures it has no physical meaning. The remaining two solutions, (e) and (g), have comparable errors and must be accepted. In the equatorial plane these two cases are similar in having the brightest region just inside the photospheric limb, while in the polar region, case (e) has a gradual diminution of intensity, and case (g) has a bright edge outside the photosphere which is superimposed upon a more rapid diminution. Case (e) has been taken to give the best approximation to the radio sun since it has the smaller number of parameters. A model with these temperatures represented by various shades of gray is shown in figure 6.

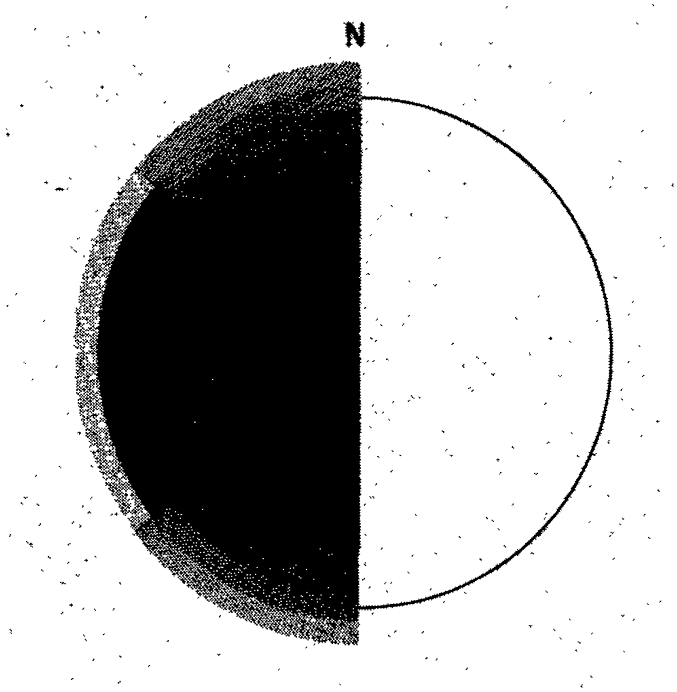

FIG. 6—Model of 10-centimetre radio sun—split along north-south axis to show relation of photosphere to temperature distribution obtained in the analysis (c.f. Table II, case (e)).

Conclusions

The early experiments in solar radio astronomy showed that radio waves originated in the direction of the sun, but gave no information on the size of the source. Calculations which were based upon an assumed emitting disk the same size as the photosphere yielded equivalent temperatures much bigger than the optical value of 6000°K., and were taken to indicate that the radio emissions were associated with the upper solar atmosphere, in particular with the solar chromosphere and the corona. These observations led Martyn to predict that solar limb brightening would be evident in the decimetre region of the spectrum. This has now been demonstrated by several observers and current problems are now shifted to the determination of the exact nature and position of the bright ring. The present work shows definitely that in the 10-centimetre region the sun is asymmetrical with pronounced bright regions on the eastern and western edges. The peak of this radiation occurs just inside the photosphere and its position should be regarded with caution since it could be very sensitive to the particular transition region between the polar and equatorial regions. In the polar regions two entirely different temperature distributions are equally satisfactory in explaining the observed curves: either a gradual darkening without a bright ring, or a more pronounced darkening with a bright edge outside the photosphere. In spite of these uncertainties, the existence of an asymmetrical sun with bright equatorial regions seems certain. This finding shows that the bright radio regions are associated with the base of the coronal equatorial streamers which were recorded at the time of this eclipse.

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