A COMPOUND INTERFEROMETER*

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ABSTRACT

The formation of a single-lobed, fan-shaped receiving pattern for a radio telescope has been accomplished by combining several individual interference patterns. The antenna described is 600 feet long 185 metres), forms a pattern 2 degrees N-S by 1.2 minutes E-W at an operating wave-length of ten centimetres, and is used for solar noise observations at the National Research Council, Ottawa. Preliminary interpretations of some of the scanning curves of the solar disk suggest that the E-W extent of the radio emissive regions associated with the sun-spots is seldom smaller than 1.5 minutes of arc.

Introduction. A compound interferometer has been constructed at the laboratories of the National Research Council in Ottawa, and is being used to study solar radio emission at a wave-length of ten centimetres. Its mode of operation is derived from the phase-sensitive interferometer first developed by Ryle (1952), and for a given linear aperture it gives the sharpest possible single-lobed fan-shaped antenna pattern (Covington and Harvey). It may be simply described as an interferometer with dissimilar elements: one is a continuous line array which produces a single-lobed pattern; the other is a four-element grating which produces a multi-lobed antenna pattern. The resultant interferometer pattern is given by a multiplication of the two individual antenna patterns, and since the two antennas are placed end-to-end in a straight line so that the phase centres do not overlap, an extra cosine interference term appears as a third multiplying factor. The performance of the antenna system may also be regarded as the selection of one of the fine lobes of the grating antenna by the single lobe produced by the continuous array, accompanied by a further sharpening of the selected grating lobe by the extra interference term.

An aerial view of the interferometer, which is located ten miles south of Ottawa at Goth Hill, is shown in figure 1. The line of antenna elements which comprise the interferometer is oriented due East-West, with an extent from tip to tip of 600 feet (185 metres). The easterly end of this length is occupied by the 150-foot horn which appears in the upper right-hand corner of figure 1. The westerly part has four parabolic cylinders, 8 feet by 10 feet, which form the grating antenna and appear in line

*Paper presented to the Spring Meeting of U.R.S.I., Washington, D.C., May 4–7, 1959. National Research Council Reprint No. 5509.

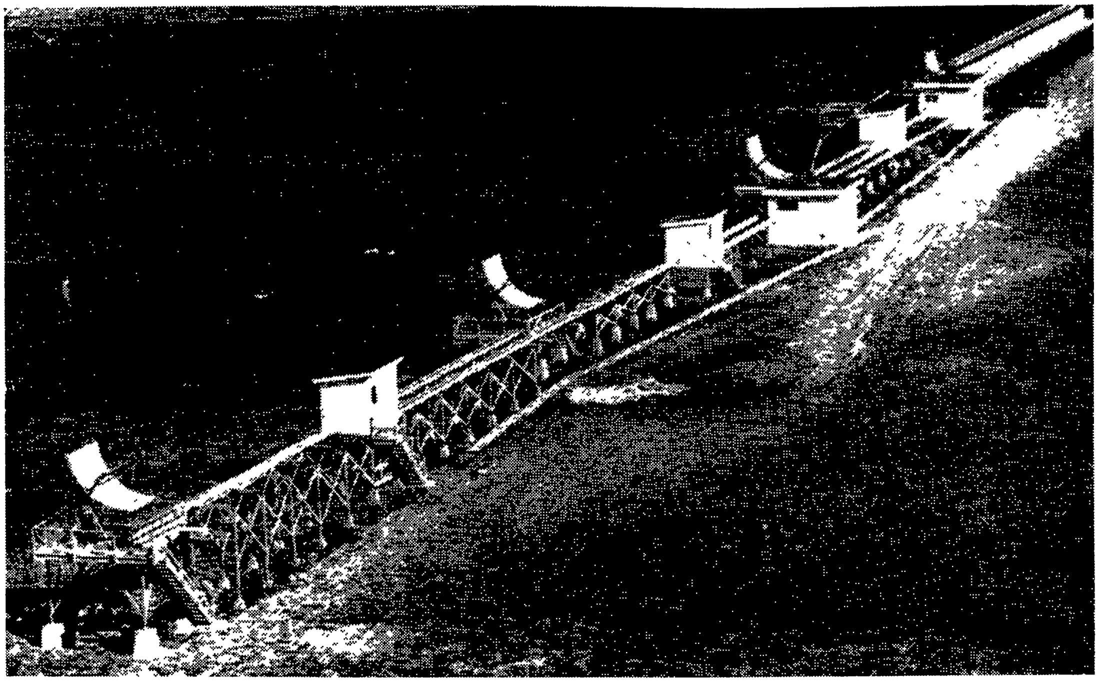

FIG. 1—Aerial view of the 10-cm. compound interferometer, 600 feet long (185 metres), in E-W direction. Upper right-hand corner shows 150-foot slotted waveguide array; lower left-hand corner shows 4-element grating antenna with gaps of 150 feet.

with the array in figure 1. The wooden trestle seen between the grating elements was constructed to support the waveguide transmission lines which provide the electrical radio-frequency connection between the elements. The transmission lines start from each element, are joined together in pairs at junctions which are housed in the small shelters with sloping roofs, and continue to the nearer flat-roofed building which houses the receiver equipment. The copper waveguide transmission lines are of rectangular cross-section, 3 inches by 1.5 inches, and have been placed in insulated wooden boxes in an attempt to keep the feeder system at a uniform temperature under various conditions of weather during the day's observations.

The various antenna elements of the interferometer may be set in elevation about the E-W axis, so that this instrument is used in the manner of a meridian telescope. Each day, near noon, the sun drifts through the receiving pattern of the instrument, which has a single lobe with a half-power width of 1.2 minutes of arc in the E-W direction, and 2 degrees in the N-S direction. Thus, with this fan-shaped antenna pattern, emission from the solar surface is examined in N-S strips. The use of travelling-wave, slotted-waveguide antennas as interferometer elements means that the primary antenna patterns, and hence the resultant pattern, may be placed at various angular positions from the celestial meridian by a suitable choice of the frequency of waves received. Four narrow-bands of frequencies, in the range 2800 mc./s. to 3000 mc./s., have been placed into service through the use of the upper and lower sidebands of two superheterodyne receivers, and enable four antenna patterns to be formed in the range of 3.5 to 7.5 degrees east of the celestial meridian.

The antenna in its present form has evolved from two earlier stages which have been described elsewhere. Initially the 150-foot slotted wave-guide array was constructed and used with a DC radiometer for studies of solar radio emission (Gruenberg 1954, Covington and Broten 1954). Although observations of the solar disk and its features with this instrument proved instructive (Covington and Broten 1954, Dodson 1954), it was very apparent that increased resolution was needed to find the size and position and to examine the structure of the still unresolved radio emissive regions. This need for increased resolution led to the development of the compound interferometer, and the construction of a two-element grating for use with the array (Covington and Broten 1957). Observations again showed that in spite of the four-fold increase in resolution there were still many unresolved radio-emissive regions associated with sun-spots. The resolution was increased by a further factor of two by the use of a four-element grating, observations with which have been carried out since May 1958 (Barber 1958, Covington 1959). Some of the results for this period will be discussed and interpreted, after the formation of the antenna pattern has been discussed.

General Theory. The derivation of the antenna pattern for the compound interferometer, as well as an indication of its mode of operation, will be made by considering the energy picked up by the two dissimilar antenna elements of a simple interferometer, designated A and B in figure 2. The incident wave-front originates from a single distant point-source, and in order to describe the thermal and quasi-thermal sources which are being investigated, it is necessary to assume that the phase of the sinusoidal vibration contains a random variable ϕ_r in addition to the usual time variable ωt, where ω is the angular frequency of the wave, and t the time. Waves of the same frequency, originating from different directions, contain other random phase terms not related to the first. The amplitudes of the voltages induced in the two antennas A and B are functions $A(\theta)$ and $B(\theta)$ of the angle θ, where this angle is between the normal to the line joining the two antennas and the incident ray. This angular displacement θ of the ray introduces equal but opposite phase changes in the two waves picked up by the two antennas. This

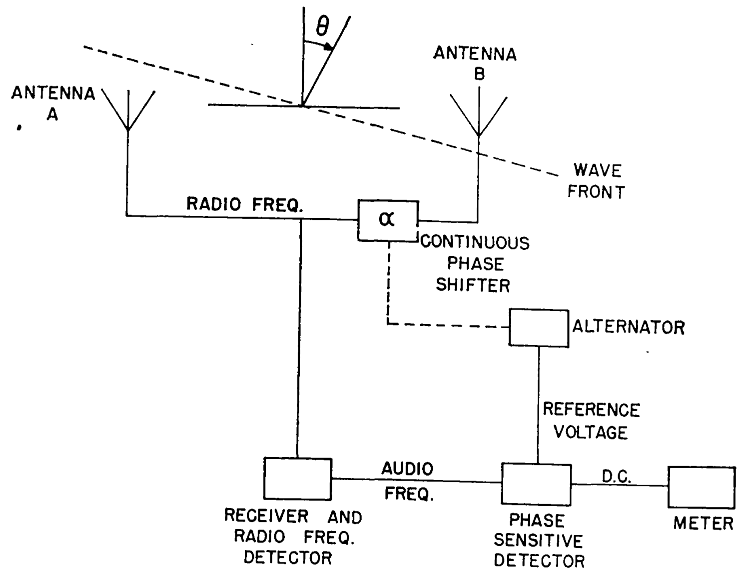

FIG. 2—Components of phase-sensitive interferometer.

phase change equals $\pi\ (D/2\lambda)\theta$, where D is the distance between the two antennas and λ is the wave-length. The two voltages travel through two transmission lines of equal length to a junction, are added together, with a loss designated by k, and then travel to the receiver. In one of the interferometer arms there is a continuously rotating phase shifter which changes the phase of one voltage with respect to the other at a very slow rate, designated by the frequency α. Thus the radio frequency voltage delivered to the receiver, expressed in complex vector notation involving the use of the previously defined variables is

$$V(\theta) = k\left[A(\theta)\exp\left(-\pi\frac{D}{2\lambda}\theta\right) + B(\theta)\exp\left(\pi\frac{D}{2\lambda}\theta + \alpha t\right)\right] \times \exp(j\omega t + \phi_r) \qquad (1)$$

The power $P(\theta)$ delivered to the receiver is found from the multiplication of $V(\theta)$ by one-half of its complex conjugate, and consists of three terms

$$P(\theta) = \frac{k^2}{2}\left[A^2(\theta) + B^2(\theta) + 2A(\theta)B(\theta)\cos\left(\pi\frac{D}{\lambda}\theta + \alpha t\right)\right] \qquad (2)$$

The first two terms consist of squared quantities independent of the low frequency α, while the last term consists of the product of three factors, and is a function of α. It is this last term, with a finite value of D, which gives rise to the compound interferometer pattern. After the received power has been amplified by the superheterodyne receiver to a suitable level and the radio frequency variations which serve as a carrier for the received power have been removed, the wanted fluctuating power represented by the last term in Eq. (2) can be separated from the steady power represented by the first two terms. This is accomplished by passing the rectified receiver output through an amplifier designed to accept only signals with the modulation frequency generated by the operation of the rotary phase shifter. After this, it is necessary to remove "*at*" from the cosine factor so that the only part remaining is one relating to the angular variable θ. This is done by passing the selected low-frequency signal through a phase-sensitive detector driven in synchronism with the rotary shifter (see figure 2). The mathematically equivalent operation performed on the fluctuating power term in Eq. (2) is a multiplication of that term by cos *at*, and subsequent expansion as a sum of two new terms. The arbitrary phase angles which arise in actual practice have been assumed to be zero, so that the compound interferometer pattern as given by the appropriate expansion term is

$$k^2 A(\theta)\, B(\theta) \cos\left(\pi \frac{D}{\lambda} \theta\right) \qquad (3)$$

A schematic of the asymmetrical interferometer elements used in the compound interferometer is shown in figure 3. The rectangle *A* represents

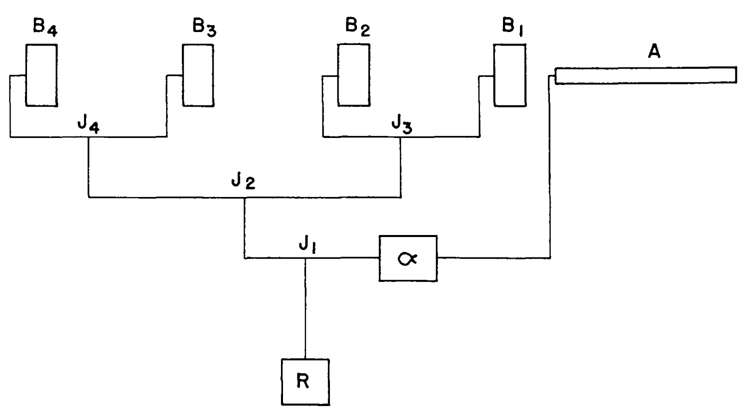

FIG. 3—Schematic of the antenna elements comprising the compound interferometer: *A*—continuous array; $B_1B_2B_3B_4$—grating elements; α—continuously rotating phase shifter; *R*—receiver; $J_1J_2J_3J_4$—junctions in waveguide transmission line.

the aperture of the long array, while the grating elements—short sections of a parabolic cylinder—are represented by B_1, B_2, B_3, and B_4, and are placed with gap separation equal to the length of the array. Waveguide transmission lines, with the rotary phase shifter placed in the transmission line coming from the array, bring the collected energy to the receiver via a series of junctions J_1, J_2, J_3, and J_4. The junctions contain removable waveguide tees or elbows so that it is possible, with a DC radiometer, to receive energy from the array, A; or, with a phase-sensitive radiometer, to use the following combinations of antennas: A with B_1, A with B_1B_2, and A with $B_1B_2B_3B_4$. The successive use of these four combinations of antennas to scan the sun within an interval of one-half hour is possible because of the beam-shifting feature of the slotted waveguide arrays, and results in the formation of the different antenna patterns at various angles from the meridian. These angles of maximum reception east of the meridian are usually defined as "squint angles".

The mathematical expressions for the resultant antenna patterns derived from the successive use of the useful combinations of elements in the grating, as outlined above, will be developed from the basic formula for the phase-sensitive interferometer given by Eq. (3). For the present, it will be assumed that the waveguide feeders are of equal length; that the various arbitrary phases in the extra interference term have been reduced to zero, and that the correct radio frequency defining the squint angle has been chosen. For convenience, in this ideal case the following substitutions are made.

$$x = \pi \frac{D}{\lambda} \theta,$$

$$A(\theta) = \frac{\sin x}{x},$$

$$B(\theta) = 1.$$

The interference term involving the B_1 element is $\cos x$,
that involving the B_2 element is $\cos 3x$,
that involving the B_3 element is $\cos 5x$,
and that involving the B_4 element is $\cos 7x$.

The resultant compound interferometer pattern is derived from the appropriate sum of the single patterns which arise from the operation of each of the grating elements as a phase-sensitive interferometer. As before, in passing through a tee junction, a voltage loss of k is involved. Thus, the power pattern $P(x)$ arising from the array operating with B_1 is given by the equation

$$P(x) = k^2 \frac{\sin x}{x} \cos x \tag{4a}$$

$$= k^2 \frac{\sin 2x}{2x};$$

from the array operating with B_1 and B_2 is given by the equation

$$P(x) = 1k^3 \frac{\sin x}{x} (\cos x + \cos 3x) \tag{4b}$$

$$= 2k^3 \frac{\sin x}{x} \cdot \cos x \cdot \cos 2x \tag{4c}$$

$$= 2k^3 \frac{\sin 4x}{4x};$$

from the array operating with $B_1B_2B_3$ and B_4, and is given by the equation

$$P(x) = 1k^4 \frac{\sin x}{x} (\cos x + \cos 3x + \cos 5x + \cos 7x) \tag{4d}$$

$$= 4k^4 \frac{\sin x}{x} \cdot \cos x \cdot \cos 2x \cdot \cos 4x \tag{4e}$$

$$= 4k^4 \frac{\sin 8x}{8x}.$$

The cosine terms in Eqs. (4a), (4b) and (4d) represent basic interference patterns which may be combined mathematically to give new cosine multiplying factors in an alternative mathematical description of the resultant antenna pattern; e.g. Eq. (4d) becomes Eq. (4e). This latter description suggests the name "compound interferometer" for this antenna, with its particular configuration of elements.

It is also apparent that a doubling of the aperture of the grating by the use of twice as many grating elements will reduce the principal lobe of the antenna pattern by one-half. This is represented mathematically by the introduction to the original antenna pattern of a single cosine multiplying factor, with amplitude $2k$; for example, Eq. (4c) becomes (4e) with the multiplier $2k \cos 4x$. The phase term of the new multiplier is twice that of the highest phase term in the product expression for the original antenna pattern. The compound interferometer patterns produced by gratings of various numbers of elements have the same functional form, namely $\sin x/x$, but have different widths of the main lobe, corresponding to the linear extent occupied by the whole antenna.

In the construction of the actual compound interferometer it was found convenient to introduce a small spacing between the end of the grating and the array. This has the effect of modifying the sidelobes so that they are predominantly negative, and in scanning the solar disk these give rise to an apparent negative value for the temperature of the sky adjacent to the sun. The theoretical pattern calculated from measured dimensions of the antenna is shown in figure 4.

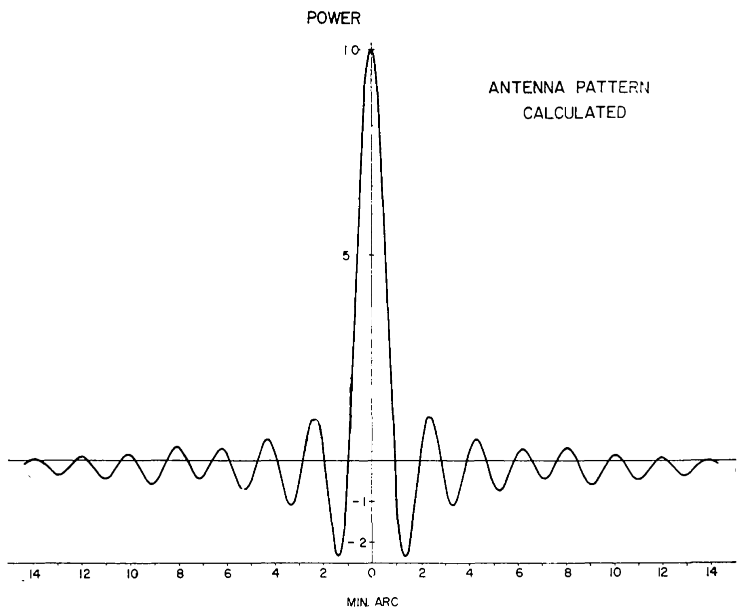

Fig. 4—Theoretical antenna pattern for the 600-foot compound interferometer.

Adjustment of the Antenna. It is impossible to observe the receiving pattern of this or any large antenna in the usual manner by recording the transmissions from a distant movable transmitter, so that complete reliance must be placed upon the quality of the components and the accuracy of adjustment. The observation of drift curves of natural point-like sources which approximate very closely the calculated antenna pattern is very fortunate, and gives confidence in the performance of the antenna. In the formation of this compound interferometer pattern, it is necessary that the maxima of the three separate elemental interferometer patterns (see Eq. (3)) be formed at the same squint angle, in order to obtain a single-lobed pattern. The position of the array pattern is controlled by the frequency of the operation, and, unfortunately, by its temperature; the grating pattern is controlled by the spacing of the elementary antennas and by the phase length of the feeders; finally, the angular position of the interference term is controlled by the phase length of the array feeder, as well as by the mechanical setting of the alternator which is driven from the rotary phase shifter and in turn drives the zero beat detector. Some of the physical factors which enter into the formation and control of these three patterns will be discussed.

The long array pattern which forms the foundation for all subsequent patterns has been shown to be very satisfactory. The grating pattern is formed by small elements so that they could be tested individually in the usual way, and later placed into the positions of the grating configuration. All these five antenna elements are connected together, by means of waveguide transmission lines and tee junctions, so that the distances from the phase centres of each of the antennas to the receiver are all equal. Referring to figure 3, it can be seen that certain sections of the lines are used in common by two grating elements, and that there is a small length of line from junction J_2 to junction J_1 to ensure that the lengths of the lines from the receiver to all phase centres are approximately equal. The phase centre of the long array is taken to be at the midpoint of the array, so that the section of waveguide from this point to the terminal of the array serves the dual purpose of slotted waveguide antenna and feeder.

The physical measurement of the feeder lengths was made without striving for an accuracy greater than that which could be realized readily. This was done because the equivalent path length includes variations related to the varying cross sections of the waveguide, as well as variations in the overall length. Both can be allowed for in the final electrical adjustments by means of phase shifters in the waveguide feeders. At first the four feeders of the grating were adjusted to integral values, but not necessarily equal integers, by using phase corrections which had been determined experimentally. These corrections were found with reference to one feeder, which is regarded as a standard, and were derived by measuring the relative standing wave pattern produced by sending a strong signal down each line, and by reflecting it back from a short which was substituted for the grating elements. The testing of a single line to each of the grating elements is made possible by replacing one or more of the tee junctions with an appropriate elbow or elbows.

There is, of course, a difference in the electrical lengths of the various arms, equal to some exact number of half-wave-lengths, which cannot be detected by means of standing waves. This gives rise to the formation of several different and unwanted antenna patterns. These half-wave-length differences can be represented analytically by the polarity (+ or −) of the elementary interference patterns generated by the array when operating with given grating elements (definitions for eqs. (4)) and the several antenna patterns calculated. Observations of actual point-like sources are required to select the single desirable pattern from the other patterns. When advantage is taken of the previous correct operation of the array operating with two grating elements, then only four of the eight combinations of half-wave-length differences need be tested in this manner. The correct pattern is given by the voltages represented by the combination of signs such as $+ + + +$ which appear in Eq. (4d), where the first sign refers to the polarity of cos x, the second sign to the polarity of cos $3x$, etc. The three undesired antenna patterns may be represented by sums of cosine terms designated by the combinations of signs: $+ + + -$, $+ + - +$, and $+ + - -$. The compound interferometer patterns arising from the correct and the three incorrect combinations of single patterns are shown in figure 5, the designated order com-

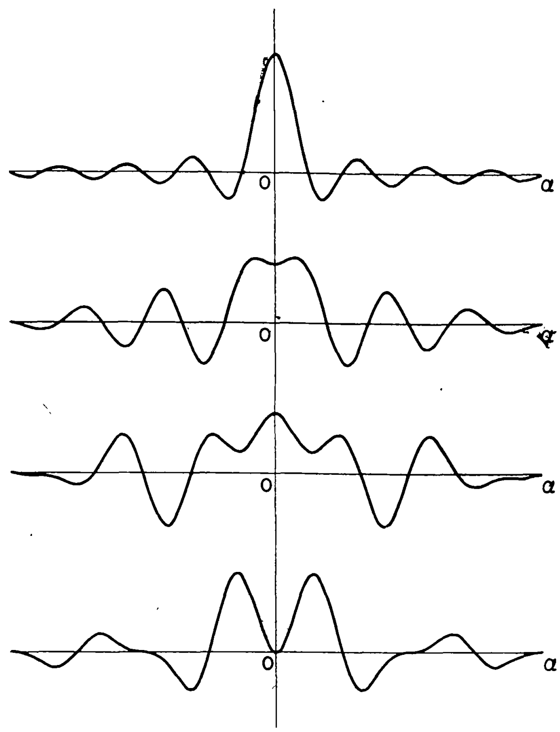

FIG. 5—Theoretical antenna patterns arising from various combinations of exact half-wave-length differences in feeder lengths to antenna elements B_3 and B_4.

mencing at the top and proceeding downwards according to the outlined scheme. As previously mentioned, the identification of the particular physical configuration of feeder lengths with the correct pattern represented by all positive signs has been possible only through the appearance of a point-like sun-spot on the solar disk. During its existence on the disk, the most undesirable pattern with the two lobes was recognized and provided the key for complete identification of the series of patterns. The pattern arising from the combination $+ + + -$ has been used, and found to produce drift-curves which are very hard to distinguish from the correct one.

The adjustments required to eliminate the arbitrary phase constants of the cosine interference terms assumed in the theory to be zero, have proven to be the most critical, and contain two factors. These are: the mechanical setting of the alternator (relative to the slowly rotating radio-frequency phase shifter) which generates the driving voltage for the zero-beat detector and the adjustment of a piece of dielectric in the waveguide line connecting the array to the receiver junction. Both of these settings give rise to fixed phases which are added together and appear in all the phase terms of the cosine interference patterns: in Eqs. (4a), (4b) and (4d). Analysis shows that when the pattern is presented in the product form, the adjustable, fixed phases appear only in the phase of the cosine term which contains the overall length. Small values of this phase term introduce an asymmetrical shift of the sidelobes of the correctly phased antenna pattern, raising those on one side while lowering those on the other side (Covington and Broten 1957). This error in the drift-curve of the solar disk introduces a corresponding raising and lowering of the value of the apparent sky emission adjacent to the solar disk, and values as small as five degrees can be detected and corrected.

The effect of differences in temperature between the various elements is important, and efforts have been made to keep important parts of the structure at the same temperature. The waveguide feeders have been placed in insulated boxes, with flexible waveguide sections placed at appropriate places to relieve stresses due to thermal expansion, as well as stresses which occurred during installation. A polyfoam cover has been placed in the opening of the horn which collects the energy for the long array, and this cover provides thermal stability for short periods of time during rapid temperature changes introduced by variable cloud cover. From summer to winter the ambient temperature ranges from +30°C. to −20°C., and the accompanying changes in the waveguide dimensions produce a shift of the squint-angle of the slotted waveguide array of about five minutes of arc. This shift is offset by introducing a frequency shift in the local oscillator, which in turn shifts the band of frequencies received, and hence the squint-angle of the antenna pattern. The exact value of the frequency shift is derived from calculations using the measured temperature at the centre of the waveguide array (Gruenberg 1954).

Seasonal changes in the dimensions of each of the feeder arms also alter the path length travelled by each of the signals picked up by the antenna elements. These changes, except for small differential changes appearing from time to time, are cancelled out in the elementary interference patterns. No attempt has been made to compensate for the residual variations related to temperature differences between grating feeders since other phase variations between grating and array were found to be more critically related to the ambient temperature and other unknown effects. The best evaluation, and hence control of this latter parameter, was finally obtained from actual measurement of the phase difference between the length of the feeder going to the array and the length of the feeder going to grating element B_1. This involved connecting the two lines together at the two antennas, A and B_1, in order to form a long loop of waveguide and to insert a signal from a generator at the midpoint of the loop. Thus, by measuring the balance between the two halves of the loop, it is possible to measure the day-to-day variations in the phase lengths of these two feeders, and hence derive a daily correction. A further temperature correction is indicated because part of the feeder which consists of one-half of the slotted array itself, is usually at a temperature elevated in comparison with that of the array feeder. This temperature difference is measured each day and a correction applied.

The principal requirement of the superheterodyne receiver in its relationship to the antenna pattern is the maintenance of the frequency of the upper and lower side-bands. This means that the frequencies of the local oscillator and of the i-f amplifier should be stable. The necessary stability of 1 part in 10^5 for the local oscillator has been achieved by coupling the low-Q cavity of the reflex klystron into a high-Q cavity, and by placing both in an oven with accurately maintained temperature. The existing i-f amplifier is of conventional design, and the possibility exists that its frequency may have drifted slightly over a long period of time. Improvements to this are possible and necessary for long-term operation. Since the antenna patterns shift in angle as a function of frequency, the bandwidth of the receiver should be as narrow as possible to have minimum blurring effect on the pattern of the antenna, and yet wide enough to pass sufficient power. A 0.5 mc./s. bandwidth has been used, which produces a 10 per cent. broadening of the calculated beamwidth. It has also been found necessary to determine accurately the phase-shift through the 15-cycle amplifier.

(*To be concluded*)

CANADIAN SCIENTISTS REPORT—XX

CONTRIBUTIONS OF DOUBLE STAR ASTRONOMY

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The study of double stars is an important part of modern astronomy, contributing essential information to our knowledge of the physical properties of stars and of stellar evolution. It is appropriate that a discussion of double stars should be included in the Canadian Scientists Report Series because Canadian astronomers have long been important contributors to this branch of astronomy. Beginning with the significant studies at the Dominion Observatory between 1907 and 1911 the discovery and investigation of spectroscopic binaries have been actively continued at the Dominion Astrophysical Observatory, and later, the David Dunlap Observatory. The most recent catalogue of such binaries lists 480 systems for which orbital details are known; of these 175 or thirty-six per cent. are credited to Canadian astronomers.

Double stars constitute an important part of the stellar population. Among stars in general one in every eighteen is seen as a double (or multiple) with the 36-inch refractor of the Lick Observatory, according to Aitken. We must however remember that a great many double stars are too far away, or the components are too close together, to be separated into visual doubles, even with powerful telescopes. Systems of small dimensions are found to be double from spectroscopic observations; Campbell, in considering such systems, estimated that at least one star in four is attended by a visible, or an invisible, companion. More recent studies tend to increase that estimate. A statistical survey of radial velocities just completed at Victoria gives as preliminary values, the following percentages of spectroscopic binaries among stars in general:

B stars	49 per cent.
A stars	55 per cent.
G stars	50 per cent.
K stars	50 per cent.
M stars	59 per cent.
All types	55 per cent.

From this table we reach the surprising conclusion that about one-half of the stars, so far studied with the spectroscope, are double!

Statistics of stars relatively very near the earth agree with the above results. Van de Kamp has listed 42 stars known to be within 16 light-years of the sun. Of these 15 are double or multiple, nearly 40 per cent.